

Summer Packet for Incoming Geometry Students

Dear Parents,

As you may be aware, Geometry is a High School credit course, and incoming Geometry students will take Geometry EOC and Math 8 STAAR at the end of the school year. Geometry can be hard to learn because it represents a new way of thinking about math. So, it could be challenging for those who skip pre-Algebra in eighth-grade math. It requires students to have a solid foundation of math taught in previous math courses, such as Math 6, Math 7, Math 8, and Algebra. Thus, your students are strongly encouraged to complete this packet by the first day of school (1 topic/day is a good strategy). Please read and discuss the instructions carefully before handing them off to them.

The packet is for incoming Geometry students at Lanier. It is used to identify their learning gaps and should serve as a review of the Algebra skills necessary for success in Geometry class. I hope that this review will keep their mind mathematically active during the summer, identify weaknesses in Algebra, if they exist, and prepare them for the challenging year ahead.

When the school year starts, the geometry classes will hit the ground running: no more than one week will be devoted to reviewing these skills and principles from Algebra 1. This assignment will give them a better idea of their readiness as they work through it. I will post the key on the first day of school. Or if they have finished the packet early and need the key to check work, please email Mr. Le at bao.le@houstonisd.org. Students will receive an extra credit grade for the completion of the assignment if they check & correct wrong answers with the key. Students will have a quiz based on the concepts covered in this packet during the second week of the school year.

Parents are requested to ensure they complete the assignment seriously as directed. Your child's success in the first semester depends on the correct completion of this packet and understanding of the concepts covered. Once they have finished the assignment, please have them explore more videos and extra practice about Geometry on Khan Academy: <https://www.khanacademy.org/math/geometry> so that they will have a great start to the new challenging school year.

Students: please be aware that **No work = No credit** will be applied and make sure that

- Your answer should be in pencil with step-by-step work shown on this packet or on your notebook.
- Your work must be neat, legible, and organized.
- NO CALCULATOR, NO SOFTWARE SOLVER should be involved in this packet.
- If your answer involves radicals or π , give an exact answer in terms of radicals or π .
- For more help, please watch tutorial videos in the following link: <https://sites.google.com/view/bao-le/algebra-1/tutorial-videos>

TOPIC 1: SOLVING MULTI-STEP EQUATIONS.

Solve each equation.

1) $\frac{19}{20} = p - 1\frac{1}{4}$

2) $b + 1 = -2\frac{1}{3}$

3) $-2 = \frac{3}{2}a$

4) $-\frac{3}{4} = \frac{3a}{7}$

5) $-2\frac{14}{15} = -2\frac{1}{5}n$

6) $\frac{5}{4} + x = \frac{3}{4}$

7) $\frac{7}{15} = 1\frac{2}{5}x$

8) $v - 2 = -1\frac{3}{5}$

9) $\frac{3}{2} = r + \frac{1}{2}$

10) $\frac{1}{3}x = -\frac{5}{12}$

11) $-7\frac{3}{8} = -2m$

12) $x - \frac{4}{7} = \frac{12}{77}$

13) $-18 - 6k = 6(1 + 3k)$

14) $5n + 34 = -2(1 - 7n)$

15) $2(4x - 3) - 8 = 4 + 2x$

16) $3n - 5 = -8(6 + 5n)$

17) $-(1 + 7x) - 6(-7 - x) = 36$

18) $-3(4x + 3) + 4(6x + 1) = 43$

19) $24a - 22 = -4(1 - 6a)$

20) $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$

Solve each multistep equation. Give your answers in the simplified form.

$$1) \frac{11}{3} \left(-\frac{1}{6}k - 2 \right) = -\frac{71}{9} + \frac{1}{2}k$$

$$2) -\frac{1}{6} \left(\frac{8}{7}x + \frac{3}{2} \right) + 8 = -\frac{239}{20} + \frac{9}{2}x$$

$$3) \frac{1}{2} + \frac{5}{2} \left(\frac{1}{2}m + 3 \right) = \frac{34}{5} + \frac{7}{5}m$$

$$4) -\frac{12}{7} + \frac{2}{5} \left(\frac{10}{3}n + \frac{1}{3} \right) = \frac{5}{7}n - \frac{296}{105}$$

$$5) \frac{23}{9} - \frac{1}{3}v = -\frac{5}{3} \left(\frac{3}{8}v - 2 \right)$$

$$6) \frac{12}{5} \left(\frac{2}{3}x - 1 \right) = -\frac{67}{60} + \frac{25}{6}x$$

$$7) \frac{10645}{504} + \frac{17}{6}p = \frac{5}{2} \left(-\frac{13}{4}p + \frac{8}{7} \right)$$

$$8) \frac{2}{7} + \frac{1}{7} \left(\frac{4}{3}x + \frac{3}{4} \right) = -\frac{6997}{588} + \frac{7}{2}x$$

$$9) \frac{49}{60} + \frac{1}{6}n = -\frac{17}{6} \left(\frac{4}{3}n + \frac{9}{5} \right)$$

$$10) -\frac{13}{4} \left(x + \frac{5}{2} \right) = -\frac{19}{24} + \frac{9}{4}x$$

$$11) \frac{10}{3} \left(\frac{12}{7}x - 8 \right) = -\frac{647}{84} + \frac{3}{2}x$$

$$12) \frac{2}{3} \left(v - \frac{23}{8} \right) + \frac{6}{5} = \frac{7}{2}v - \frac{149}{30}$$

$$13) \frac{33}{8}a - \frac{43}{40} = -\frac{12}{5} \left(-\frac{11}{6}a - \frac{1}{8} \right)$$

$$14) -\frac{7}{4} \left(n + \frac{6}{7} \right) - 3 = -\frac{119}{24} - \frac{15}{7}n$$

$$15) \frac{7}{3}r - \frac{877}{216} = \frac{11}{6} \left(\frac{11}{6}r - \frac{3}{4} \right) - 2$$

$$16) \frac{14}{3} \left(r + \frac{7}{8} \right) = \frac{145}{48} + \frac{11}{6}r$$

$$17) -\frac{3}{2}k - \frac{711}{20} = \frac{21}{5} \left(4k + \frac{1}{4} \right)$$

$$18) \frac{9}{2} \left(-\frac{23}{6}x - \frac{25}{8} \right) = \frac{2817}{112} - 2x$$

$$19) -\frac{5}{8}x - \frac{23}{7} \left(\frac{25}{6}x - \frac{14}{5} \right) = \frac{105821}{5880} - 2x$$

$$20) -\frac{7}{4}m - \frac{3233}{490} = -\frac{23}{7} \left(\frac{3}{7}m + \frac{9}{5} \right)$$

Solve each absolute value equation.

1) $9 + 6|p + 6| = 33$

2) $-5|x + 6| + 10 = -65$

3) $1 + 10\left|\frac{b}{6}\right| = 16$

4) $8|x - 3| + 3 = 99$

5) $5|x + 6| - 3 = 62$

6) $6\left|\frac{x}{7}\right| + 10 = 16$

7) $-9 + 5|10 + v| = 71$

8) $5|a + 10| + 5 = 75$

9) $9\left|\frac{x}{4}\right| - 9 = 0$

10) $9 + 3|k - 4| = 51$

11) $4 + 7|m - 4| = 39$

12) $3 - 5|2x| = -17$

13) $8\left|\frac{v}{10}\right| - 7 = 1$

14) $|n - 8| - 10 = -8$

15) $5|n - 1| - 2 = 13$

16) $-2 + 8\left|\frac{a}{9}\right| = 6$

17) $3|6 + k| - 3 = 24$

18) $2\left|\frac{a}{3}\right| + 6 = 8$

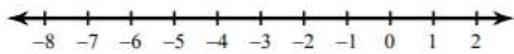
19) $|6x| + 6 = 24$

20) $4|b + 7| + 1 = 29$

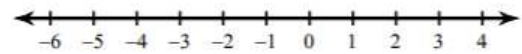
TOPIC 2: SOLVING MULTI-STEP INEQUALITIES.

Solve each inequality and graph its solution.

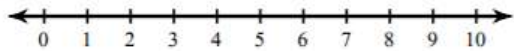
1) $3 < -5n + 2n$



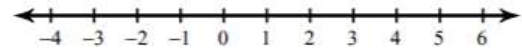
2) $6x + 2 + 6x < 14$



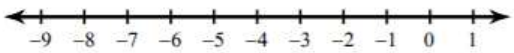
3) $-p - 4p > -10$



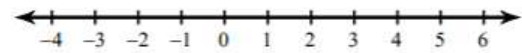
4) $18 \geq 5k + 4k$



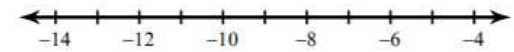
5) $9 \geq -2m + 2 - 3$



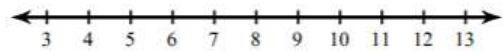
6) $-3 - 6(4x + 6) > -111$



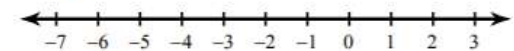
7) $6 - 4(6n + 7) \geq 122$



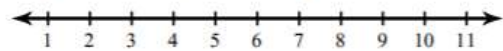
8) $-138 \geq -6(6b - 7)$



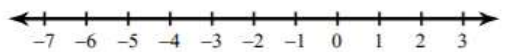
9) $167 < 6 + 7(2 - 7r)$



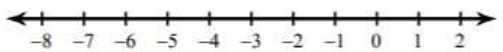
10) $5(6 + 3r) + 7 \geq 127$



11) $-8x + 2x - 16 < -5x + 7x$



12) $-1 - 6x - 6 > -11 - 7x$



TOPIC 3: SOLVING LITERAL EQUATIONS.

Solve each equation for the indicated variable.

1) $g = 6x$, for x

2) $u = 2x - 2$, for x

3) $z = m - x$, for x

4) $g = ca$, for a

5) $u = x - k$, for x

6) $g = c + x$, for x

7) $u = \frac{k}{a}$, for a

8) $g = xc$, for x

9) $12am = 4$, for a

10) $-3x + 2c = -3$, for x

11) $am = n + p$, for a

12) $u = \frac{ak}{b}$, for a

13) $a - c = d - r$, for a

14) $xm = np$, for x

Solve each equation for the indicated variable.

1) $x + m = yx + np$, for x

2) $u = \frac{ak + v}{aw}$, for a

3) $x + m = yx - np$, for x

4) $kx = vw + yx$, for x

5) $am = n - p + ba$, for a

6) $ma = pn + ba$, for a

7) $k - a = wv + ba$, for a

8) $z = \frac{am + p}{an}$, for a

9) $c + x = rdyx$, for x

10) $k - a = w - v + ba$, for a

11) $m + x = p + n + yx$, for x

12) $m - x = n + p + yx$, for x

13) $g = ac + ad + r$, for a

14) $a + m = n - p + ba$, for a

15) $cx = dr + yx$, for x

16) $cx = r + d + yx$, for x

17) $g = \frac{ac + r}{ad}$, for a

18) $c - x = d - r + yx$, for x

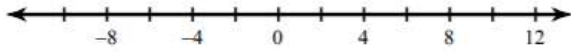
19) $xk = v + w + yx$, for x

20) $x + k = w - v + yx$, for x

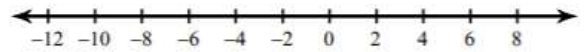
TOPIC 4: SOLVING COMPOUND INEQUALITIES.

Solve each compound inequality and graph its solution.

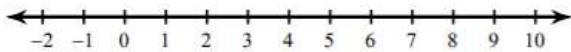
1) $m - 2 < -8$ or $\frac{m}{8} > 1$



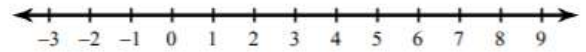
2) $-1 < 9 + n < 17$



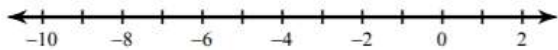
3) $2x < 10$ or $\frac{x}{2} \geq 3$



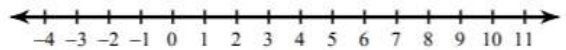
4) $x + 8 \geq 9$ and $\frac{x}{7} \leq 1$



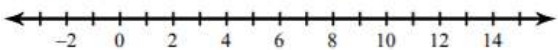
5) $-3 \leq \frac{p}{2} < 0$



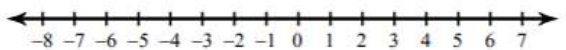
6) $r + 5 \geq 12$ or $\frac{r}{9} < 0$



7) $7v - 5 \geq 65$ or $-3v - 2 \geq -2$



8) $-10b + 3 \leq -37$ or $3b - 10 \leq -25$



TOPIC 5: SOLVING SYSTEM OF EQUATIONS

Solve each system by elimination.

1) $-4x - 2y = -12$
 $4x + 8y = -24$

2) $4x + 8y = 20$
 $-4x + 2y = -30$

3) $x - y = 11$
 $2x + y = 19$

4) $-6x + 5y = 1$
 $6x + 4y = -10$

5) $-2x - 9y = -25$
 $-4x - 9y = -23$

6) $8x + y = -16$
 $-3x + y = -5$

7) $-6x + 6y = 6$
 $-6x + 3y = -12$

8) $7x + 2y = 24$
 $8x + 2y = 30$

9) $5x + y = 9$
 $10x - 7y = -18$

10) $-4x + 9y = 9$
 $x - 3y = -6$

11) $-3x + 7y = -16$
 $-9x + 5y = 16$

12) $-7x + y = -19$
 $-2x + 3y = -19$

Solve each system by substitution.

1) $y = 6x - 11$
 $-2x - 3y = -7$

2) $2x - 3y = -1$
 $y = x - 1$

3) $y = -3x + 5$
 $5x - 4y = -3$

4) $-3x - 3y = 3$
 $y = -5x - 17$

5) $y = -2$
 $4x - 3y = 18$

6) $y = 5x - 7$
 $-3x - 2y = -12$

7) $-4x + y = 6$
 $-5x - y = 21$

8) $-7x - 2y = -13$
 $x - 2y = 11$

9) $-5x + y = -2$
 $-3x + 6y = -12$

10) $-5x + y = -3$
 $3x - 8y = 24$

Solve each system by graphing.

$$\begin{aligned} 1) \quad & -4 = -x - 2y \\ & 0 = -6 - 3y - \frac{15}{2}x \end{aligned}$$

$$\begin{aligned} 2) \quad & -\frac{3}{2}x = -4 - 2y \\ & 16 - 4y - 3x = 0 \end{aligned}$$

$$\begin{aligned} 3) \quad & 0 = -3y - 12 - 2x \\ & -6 = -3y + 4x \end{aligned}$$

$$\begin{aligned} 4) \quad & 3x + y = 4 \\ & -2y + 4x = 2 \end{aligned}$$

$$\begin{aligned} 5) \quad & 6x = 8y + 8 \\ & 0 = 3x - 4y - 4 \end{aligned}$$

$$\begin{aligned} 6) \quad & 0 = x + 2 - y \\ & x = 4 + 4y \end{aligned}$$

$$\begin{aligned} 7) \quad & -8x + 2 + 2y = 0 \\ & y - x = 2 \end{aligned}$$

$$\begin{aligned} 8) \quad & 6x = -4 - 4y \\ & -3x - 4 - 2y = 0 \end{aligned}$$

$$\begin{aligned} 9) \quad & 4x + 6 = -6y \\ & 0 = -12 + 3y + 7x \end{aligned}$$

$$\begin{aligned} 10) \quad & x = 4y + 12 \\ & -4y = -4 - 5x \end{aligned}$$

$$\begin{aligned} 11) \quad & -3 - x - 3y = 0 \\ & 4 = y + 2x \end{aligned}$$

$$\begin{aligned} 12) \quad & y - 1 = -x \\ & -4 + x = 2y \end{aligned}$$

$$\begin{aligned} 13) \quad & y - 1 + \frac{5}{2}x = 0 \\ & 0 = 2y + x + 6 \end{aligned}$$

$$\begin{aligned} 14) \quad & 4y - x = -12 \\ & 3y - \frac{21}{4}x = 9 \end{aligned}$$

$$\begin{aligned} 15) \quad & -x = -4 - y \\ & 0 = -5x - y + 2 \end{aligned}$$

$$\begin{aligned} 16) \quad & 3y = x + 12 \\ & -\frac{6}{5} - \frac{3}{5}y = x \end{aligned}$$

$$\begin{aligned} 17) \quad & 16 = 4y - 7x \\ & 0 = 6 + 6y - 3x \end{aligned}$$

$$\begin{aligned} 18) \quad & 0 = -2x - y + 1 \\ & 2y + x = -4 \end{aligned}$$

$$\begin{aligned} 19) \quad & -3y = -9 + 15x \\ & 3 - x + y = 0 \end{aligned}$$

$$\begin{aligned} 20) \quad & 4 + 10x + 4y = 0 \\ & 2y = -x + 6 \end{aligned}$$

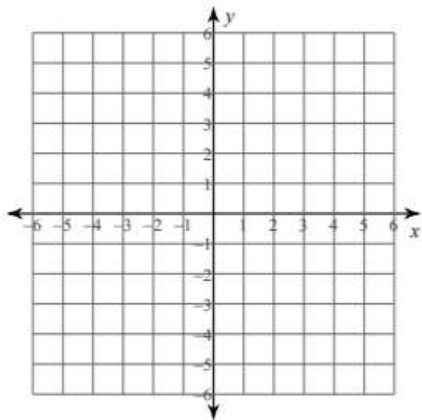
Solving system of equations word problems

- 1) The sum of the digits of a certain two-digit number is 9. When you reverse its digits you decrease the number by 63. Find the number.
- 2) The senior classes at High School A and High School B planned separate trips to the state fair. The senior class at High School A rented and filled 5 vans and 1 bus with 101 students. High School B rented and filled 1 van and 5 buses with 217 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?
- 3) Kayla and Ryan are selling cookie dough for a school fundraiser. Customers can buy packages of sugar cookie dough and packages of gingerbread cookie dough. Kayla sold 9 packages of sugar cookie dough and 6 packages of gingerbread cookie dough for a total of \$165. Ryan sold 6 packages of sugar cookie dough and 12 packages of gingerbread cookie dough for a total of \$270. What is the cost each of one package of sugar cookie dough and one package of gingerbread cookie dough?
- 4) The sum of the digits of a certain two-digit number is 15. When you reverse its digits you increase the number by 27. Find the number.
- 5) New York City is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 3 vans and 12 buses with 546 students. High School B rented and filled 9 vans and 8 buses with 490 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?
- 6) A plane traveled 720 miles to Houston and back. The trip there was with the wind. It took 8 hours. The trip back was into the wind. The trip back took 24 hours. What is the speed of the plane in still air? What is the speed of the wind?
- 7) The senior classes at High School A and High School B planned separate trips to the local amusement park. The senior class at High School A rented and filled 4 vans and 1 bus with 47 students. High School B rented and filled 13 vans and 9 buses with 285 students. Each van and each bus carried the same number of students. Find the number of students in each van and in each bus.
- 8) Nicole's school is selling tickets to a choral performance. On the first day of ticket sales the school sold 11 adult tickets and 7 child tickets for a total of \$198. The school took in \$165 on the second day by selling 1 adult ticket and 14 child tickets. Find the price of an adult ticket and the price of a child ticket.

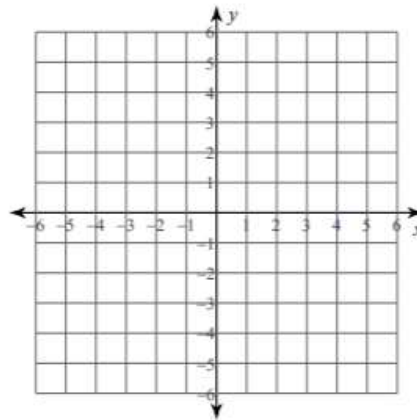
TOPIC 6: GRAPHING LINEAR FUNCTIONS

Sketch the graph of each line.

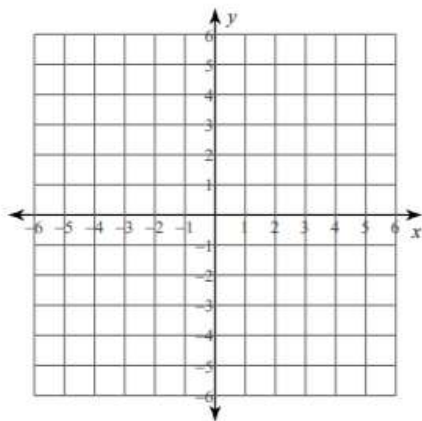
1) $y = \frac{7}{2}x - 2$



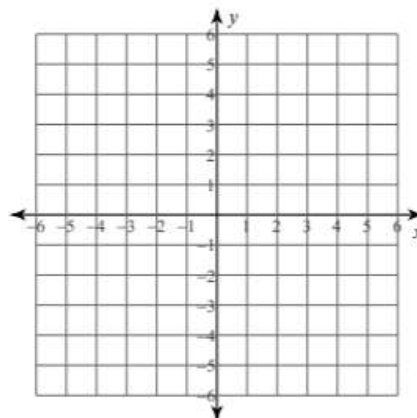
2) $y = -6x + 3$



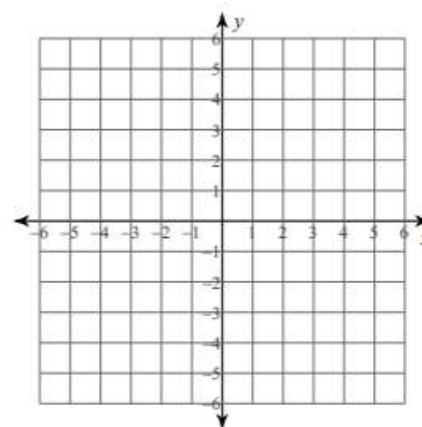
3) $y = -5$



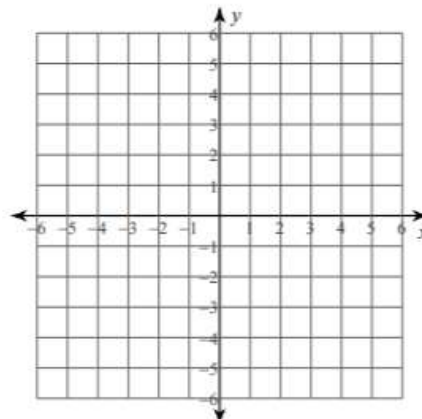
4) $y = \frac{6}{5}x + 1$



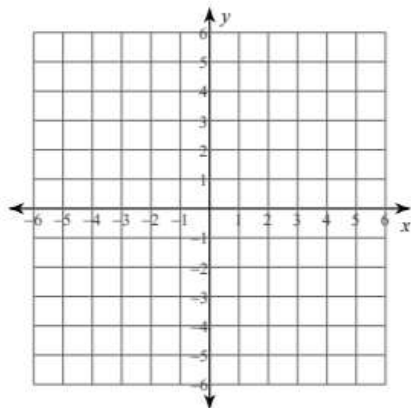
5) $y = \frac{1}{4}x + 2$



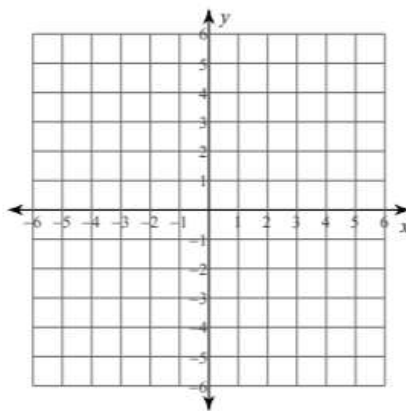
6) $x = 5$



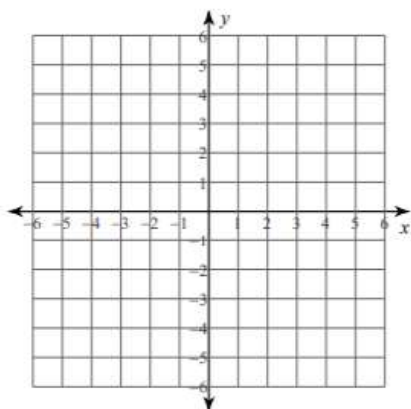
7) $y = \frac{5}{3}x$



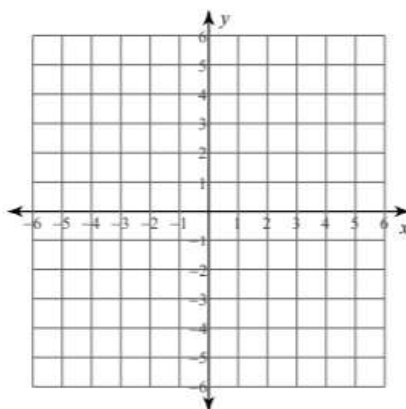
8) $x = 0$



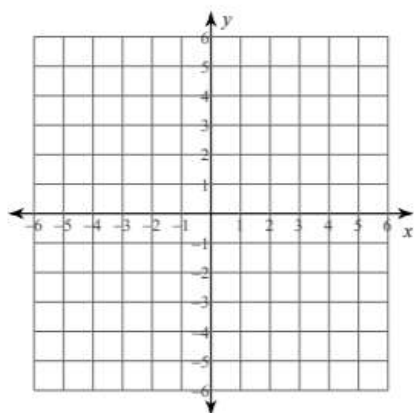
9) $y = -\frac{1}{3}x + 3$



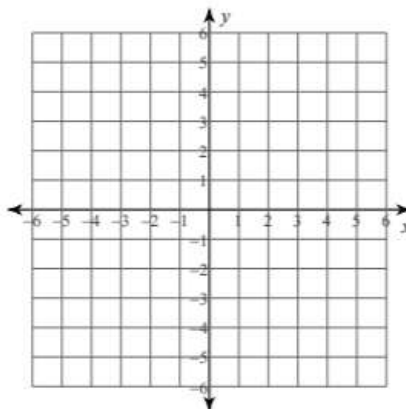
10) $y = \frac{1}{5}x - 4$



11) $y = \frac{1}{2}x - 2$



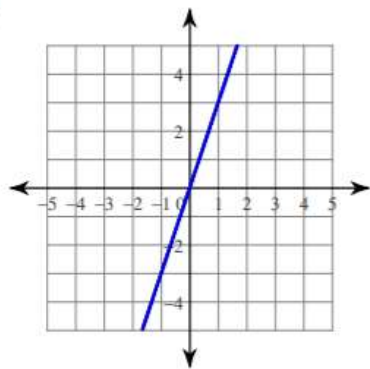
12) $y = 2x + 5$



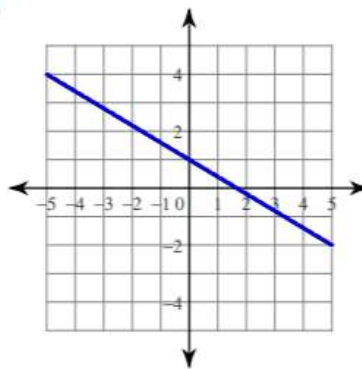
TOPIC 7: WRITING LINEAR FUNCTIONS

Write the slope-intercept form of the equation of each line.

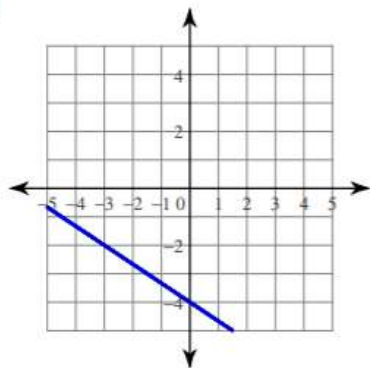
1)



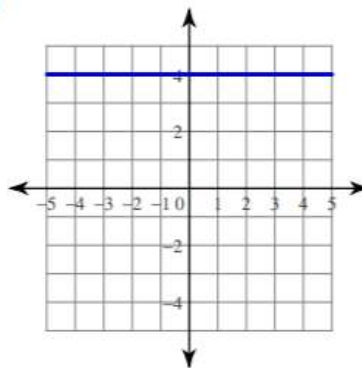
2)



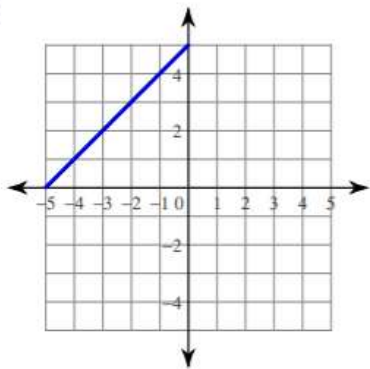
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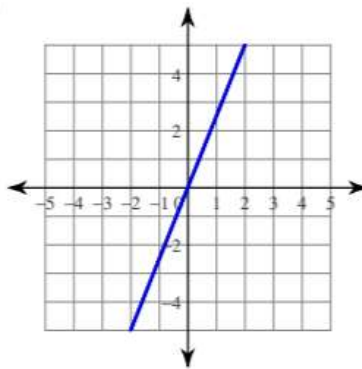
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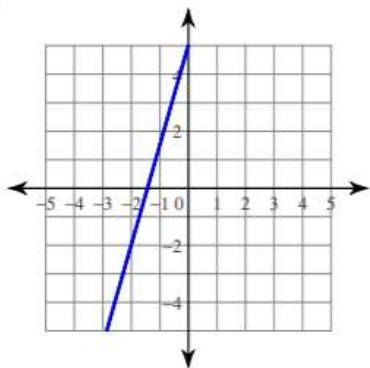
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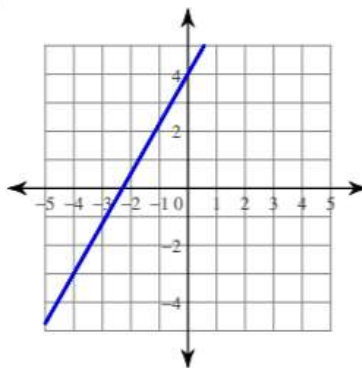
6)



7)



8)



Write the slope-intercept form of the equation given a table

13.

X	Y
-2	5
-1	4
0	3
1	2
2	1

Equation:

14.

X	Y
-2	1
-1	2
0	3
1	4
2	5

Equation:

15.

X	Y
-2	-5
-1	-3
0	-1
1	1
2	3

Equation:

16.

X	Y
-2	9
-1	6
0	3
1	0
2	-3

Equation:

17.

X	Y
-4	1
-2	4
0	7
2	10
4	13

Equation:

18.

X	Y
-10	5
-5	4
0	3
5	2
10	1

Equation:

19.

X	Y
-6	5
-3	3
0	1
3	-1
6	-3

Equation:

20.

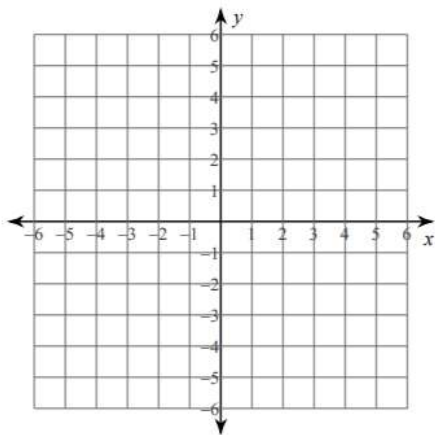
X	Y
-4	-8
-2	-4
0	0
2	4
4	8

Equation:

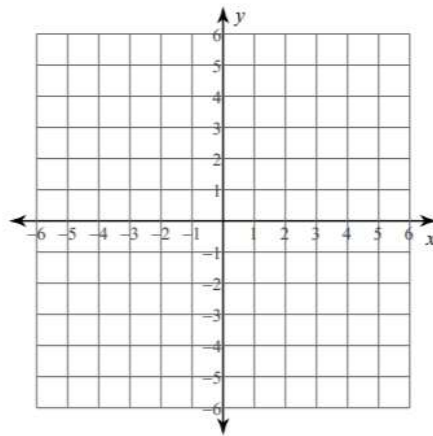
TOPIC 8: GRAPHING LINEAR INEQUALITIES

Sketch the graph of each linear inequality.

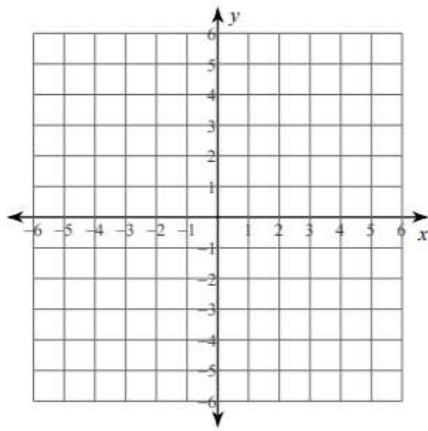
1) $y \geq -3x + 4$



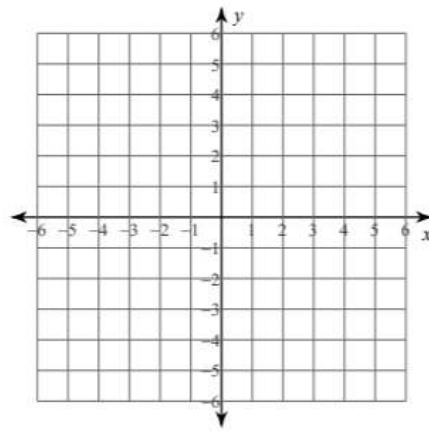
2) $y \leq \frac{3}{5}x - 5$



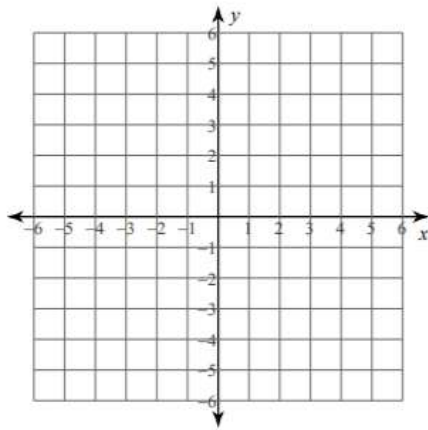
3) $y > -x - 5$



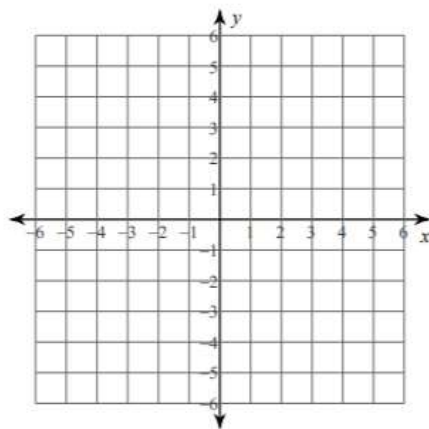
4) $y > -4$



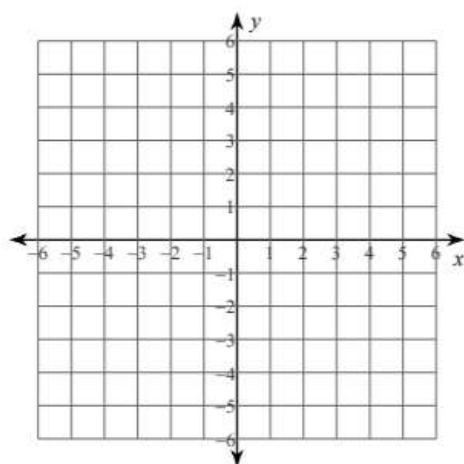
5) $y > 2x - 5$



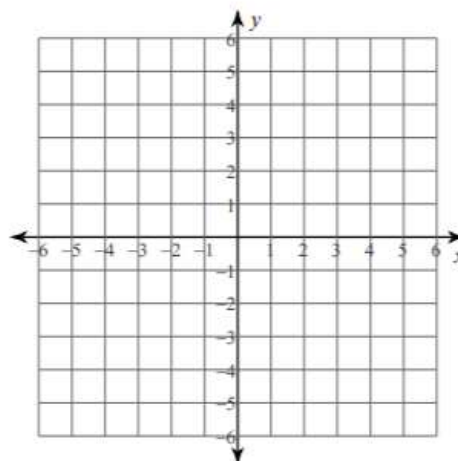
6) $y \geq \frac{7}{4}x + 2$



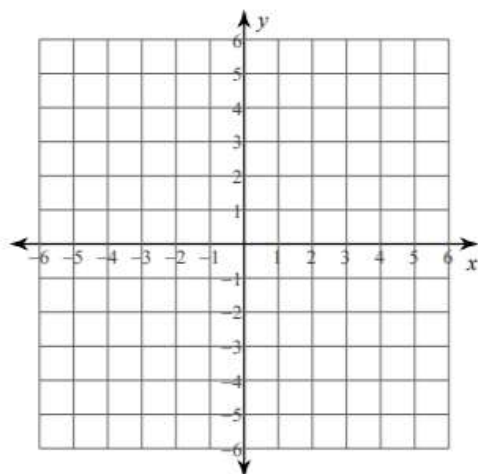
7) $x < -5$



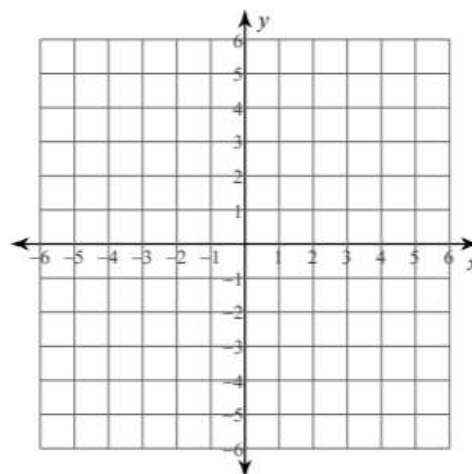
8) $y \leq \frac{4}{3}x - 4$



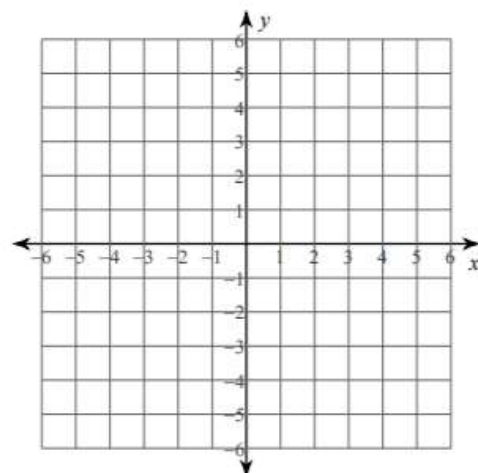
9) $3x - 2y < 10$



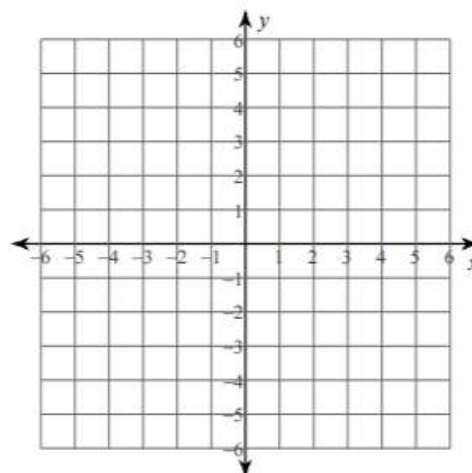
10) $5x - 3y \leq -15$



11) $y \geq 4$



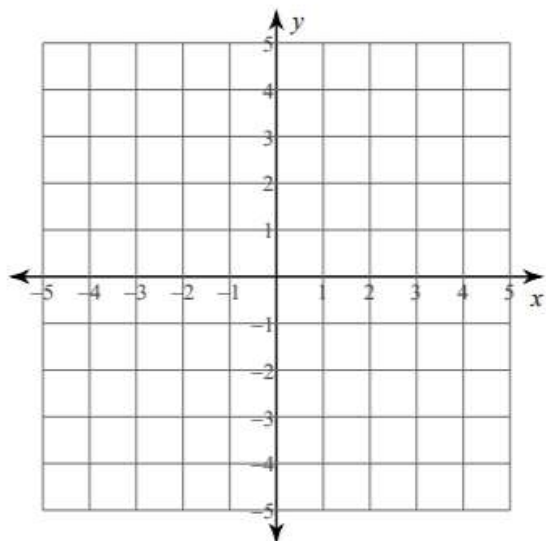
12) $x - y > 2$



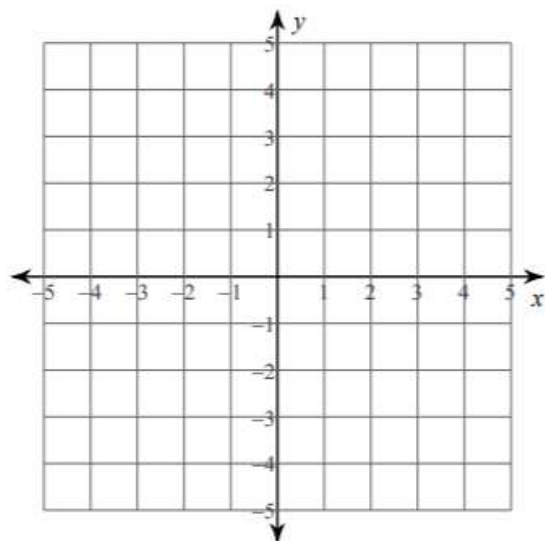
TOPIC 9: SYSTEM OF LINEAR INEQUALITIES

Sketch the solution to each system of inequalities.

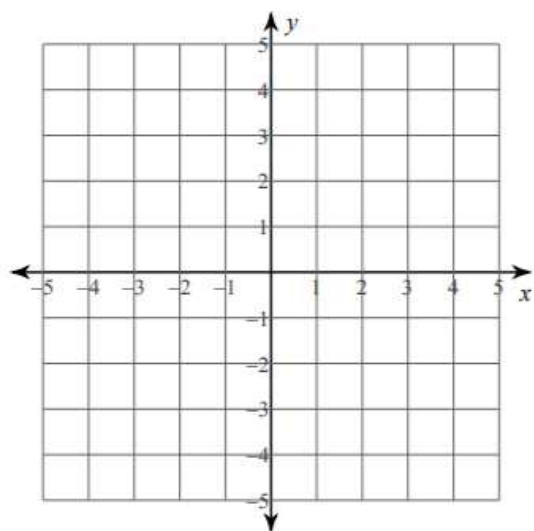
1) $y \leq -x - 2$
 $y \geq -5x + 2$



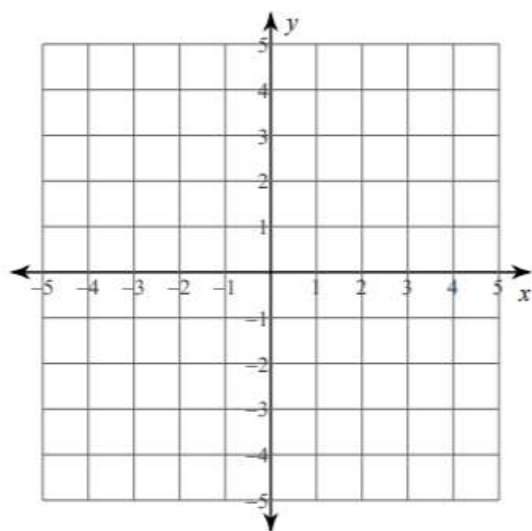
2) $y > -x - 2$
 $y < -5x + 2$



3) $y \leq \frac{1}{2}x + 2$
 $y < -2x - 3$

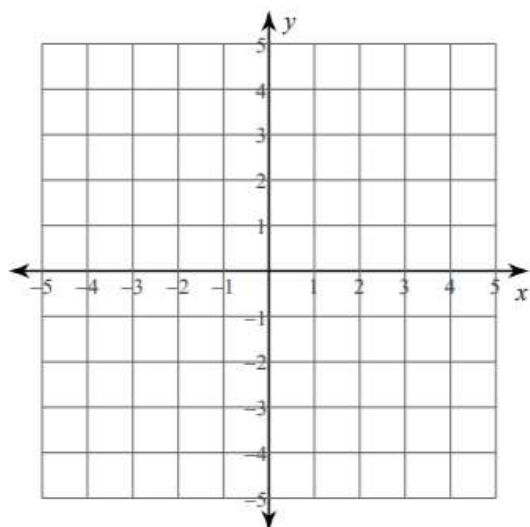


4) $x \leq -3$
 $y < \frac{5}{3}x + 2$



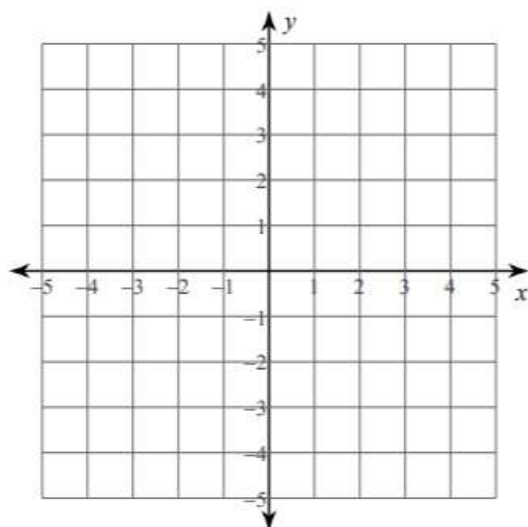
$$5) \ y \leq -\frac{5}{2}x - 2$$

$$y < -\frac{1}{2}x + 2$$



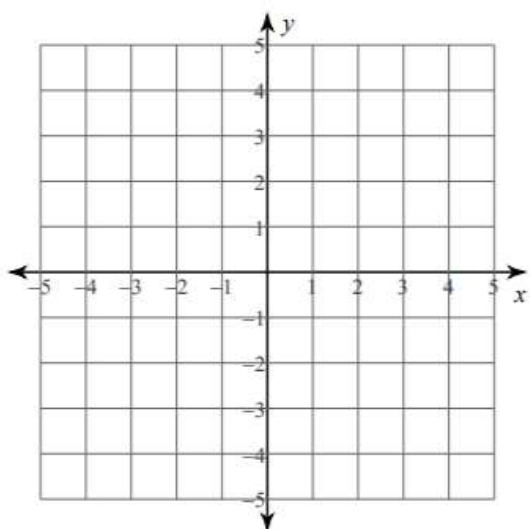
$$6) \ y \geq \frac{2}{3}x + 3$$

$$y > -\frac{4}{3}x - 3$$



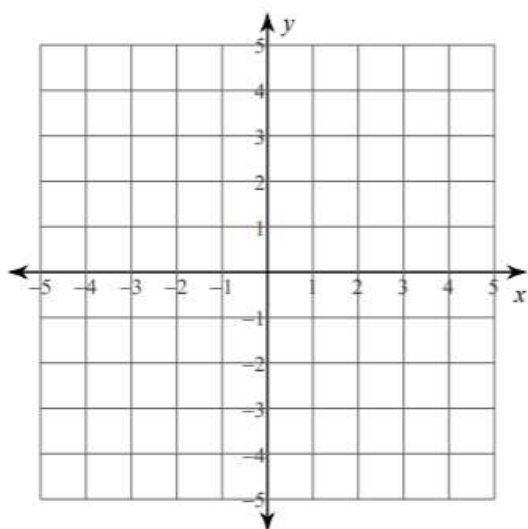
$$7) \ 4x + y < 2$$

$$y > -2$$



$$8) \ 3x + 2y \geq -2$$

$$x + 2y \leq 2$$



TOPIC 10: WRITE LINEAR EQUATIONS

Write the slope-intercept form of the equation of each line.

1) $3x - 2y = -16$

2) $13x - 11y = -12$

3) $9x - 7y = -7$

4) $x - 3y = 6$

5) $6x + 5y = -15$

6) $4x - y = 1$

7) $11x - 4y = 32$

8) $11x - 8y = -48$

Write the standard form of the equation of the line through the given point with the given slope.

9) through: $(1, 2)$, slope = 7

10) through: $(3, -1)$, slope = -1

11) through: $(-2, 5)$, slope = -4

12) through: $(3, 5)$, slope = $\frac{5}{3}$

TOPIC 11: PARALLEL & PERPENDICULAR LINES

Write the slope-intercept form of the equation of the line described.

1) through: $(5, 2)$, parallel to $y = \frac{4}{5}x$

2) through: $(-3, 3)$, parallel to $y = -\frac{7}{3}x + 2$

3) through: $(-2, -2)$, parallel to $y = \frac{5}{2}x + 1$

4) through: $(-4, 2)$, parallel to $y = \frac{3}{4}x + 2$

5) through: $(-2, -2)$, parallel to $y = -\frac{3}{7}x + 2$

6) through: $(2, -3)$, parallel to $y = -x - 4$

7) through: $(-4, -1)$, parallel to $x = 0$

8) through: $(5, 0)$, parallel to $y = -\frac{4}{5}x - 5$

9) through: $(1, 0)$, parallel to $y = -\frac{1}{4}x + 4$

10) through: $(3, -4)$, parallel to $y = -4x + 3$

11) through: $(-1, 4)$, perp. to $y = \frac{1}{2}x + 1$

12) through: $(5, -2)$, perp. to $y = \frac{5}{7}x - 2$

13) through: $(5, -2)$, perp. to $x = 0$

14) through: $(1, 2)$, perp. to $y = -x + 4$

15) through: $(3, 2)$, perp. to $y = -\frac{3}{7}x + 5$

16) through: $(-2, 5)$, perp. to $y = \frac{2}{7}x + 3$

17) through: $(2, -2)$, perp. to $y = 5$

18) through: $(-4, 0)$, perp. to $y = -4$

19) through: $(-2, -2)$, perp. to $y = 2x - 2$

20) through: $(2, -4)$, perp. to $y = \frac{1}{2}x + 4$

TOPIC 12: EVALUATE EXPRESSIONS

Evaluate each using the values given.

1) $y \div 2 + x$; use $x = 1$, and $y = 2$

2) $a - 5 - b$; use $a = 10$, and $b = 4$

3) $p^2 + m$; use $m = 1$, and $p = 5$

4) $y + 9 - x$; use $x = 1$, and $y = 3$

5) $m + p \div 5$; use $m = 1$, and $p = 5$

6) $y^2 - x$; use $x = 7$, and $y = 7$

7) $z(x + y)$; use $x = 6$, $y = 8$, and $z = 6$

8) $x + y + y$; use $x = 9$, and $y = 10$

9) $p^3 + 10 + m$; use $m = 9$, and $p = 3$

10) $6q + m - m$; use $m = 8$, and $q = 3$

11) $p^2m \div 4$; use $m = 4$, and $p = 7$

12) $y - (z + z^2)$; use $y = 10$, and $z = 2$

13) $z - (y \div 3 - 1)$; use $y = 3$, and $z = 7$

14) $(y + x) \div 2 + x$; use $x = 1$, and $y = 1$

Evaluate each using the values given.

1) $\left(-\frac{4}{p-r}\right)(1 - ((-4) + q))$; use $p = 1\frac{1}{3}$, $q = 1$, and $r = \frac{1}{3}$

2) $|z^2| - 5xy$; use $x = -\frac{4}{3}$, $y = -2\frac{1}{3}$, and $z = \frac{2}{3}$

3) $yx + x - \left(-\frac{4}{z}\right)^3$; use $x = \frac{1}{2}$, $y = \frac{1}{2}$, and $z = -\frac{5}{3}$

4) $(z)\left(x - -\frac{3}{z} - 5 - z\right)$; use $x = 2\frac{5}{6}$, and $z = -2$

5) $(m)\left(\frac{|m| + p}{n + 6}\right)$; use $m = 2\frac{1}{2}$, $n = \frac{3}{2}$, and $p = 1\frac{1}{2}$

6) $(yz)((y - x)^2 + 1)$; use $x = -\frac{1}{4}$, $y = -\frac{3}{2}$, and $z = -1$

7) $(4)(1 - p) + m^3 + 5$; use $m = \frac{2}{3}$, and $p = 1$

8) $-\frac{6xz}{y} + x + 6$; use $x = -\frac{3}{5}$, $y = \frac{1}{2}$, and $z = -6$

9) $(5)\left(k - (5)\left(\frac{h}{2 - k}\right)\right)$; use $h = -\frac{4}{3}$, and $k = -1\frac{2}{3}$

10) $(z - 5 + z - x)(x - 2)$; use $x = -2$, and $z = -\frac{5}{3}$

TOPIC 13: ADD, SUBTRACT, MULTIPLY POLYNOMIALS

Simplify each expression.

1) $(6k - 7k^2) + (4k^2 + 6)$

2) $(8 - 5k^2) - (7k^2 + 5)$

3) $(2x^2 + 5x) + (7 + 5x^2 - x)$

4) $(5p^2 - 8) - (3p + 4 - 5p^2)$

5) $(6n - 4n^4 + 5n^3) + (n^3 - 6n^4 + 7n)$

6) $(6x^4 - x^3 + 5) - (2x^4 + 3x^3 - 1)$

Multiplying Polynomials: Find each product.

7) $(5p + 3)(8p + 7)$

8) $(3a + 3)(3a - 2)$

9) $(3x + 5)(7x - 4)$

10) $(4x + 5)(4x + 6)$

11) $(k - 2)(6k + 1)$

12) $(4m + 2)(4m + 5)$

13) $(5n + 4)(4n^2 + 2n - 4)$

14) $(5b - 6)(5b^2 + 4b - 2)$

15) $(3x + 4)(5x^2 - 6x - 6)$

16) $(8x + 2)(3x^2 + 4x - 5)$

17) $(8n^2 + n - 4)(6n^2 - 6n - 4)$

18) $(6x^2 + 6x + 2)(3x^2 + 5x + 7)$

TOPIC 14: DIVIDING POLYNOMIALS

Divide.

1) $(m^2 - 7m - 11) \div (m - 8)$

2) $(n^2 - n - 29) \div (n - 6)$

3) $(n^2 + 10n + 18) \div (n + 5)$

4) $(k^2 - 7k + 10) \div (k - 1)$

5) $(n^2 - 3n - 21) \div (n - 7)$

6) $(a^2 - 28) \div (a - 5)$

7) $(r^2 + 14r + 38) \div (r + 8)$

8) $(x^2 + 5x + 3) \div (x + 6)$

9) $(2x^2 - 17x - 38) \div (2x + 3)$

10) $(42x^2 - 33) \div (7x + 7)$

TOPIC 15: SIMPLIFY RATIONAL EXPRESSIONS

Simplify each expression.

$$1) -\frac{36x^3}{42x^2}$$

$$2) \frac{16r^2}{16r^3}$$

$$3) \frac{16p^2}{28p}$$

$$4) \frac{32n^2}{24n}$$

$$5) -\frac{70n^2}{28n}$$

$$6) \frac{15n}{30n^3}$$

$$7) \frac{2r-4}{r-2}$$

$$8) \frac{45}{10a-10}$$

$$9) \frac{x-4}{3x^2-12x}$$

$$10) \frac{15a-3}{24}$$

$$11) \frac{v-5}{v^2-10v+25}$$

$$12) \frac{x+6}{x^2+5x-6}$$

TOPIC 16: SIMPLIFY RADICAL EXPRESSIONS

Simplify. Don't Forget: Never leave a radical in the denominator.

1) $\frac{\sqrt{4}}{\sqrt{12}}$

2) $\frac{\sqrt{12}}{\sqrt{4}}$

3) $\frac{\sqrt{8}}{\sqrt{10}}$

4) $\frac{\sqrt{15}}{\sqrt{9}}$

5) $\frac{\sqrt{8}}{\sqrt{50}}$

6) $-3\sqrt{6} \cdot -4\sqrt{15}$

7) $5\sqrt{5} \cdot \sqrt{15}$

8) $\sqrt{6m^2} \cdot \sqrt{6m}$

9) $\sqrt{4} \cdot -5\sqrt{2}$

10) $\sqrt{5b^3} \cdot \sqrt{15b^3}$

Simplify.

11) $6\sqrt{180n}$

12) $-8\sqrt{72x^3}$

13) $\sqrt{50m^3n^3}$

14) $-\sqrt{12}$

15) $\sqrt{45v^4}$

16) $-8\sqrt{96u^2v^2}$

17) $8\sqrt{12u^3v^2}$

18) $\sqrt{175}$

19) $\sqrt{98xy^2z}$

20) $\sqrt{45xy^2z^4}$

TOPIC 17: FACTORING POLYNOMIALS BY GROUPING

Factor each completely.

1) $8r^3 - 64r^2 + r - 8$

2) $12p^3 - 21p^2 + 28p - 49$

3) $12x^3 + 2x^2 - 30x - 5$

4) $6v^3 - 16v^2 + 21v - 56$

5) $63n^3 + 54n^2 - 105n - 90$

6) $21k^3 - 84k^2 + 15k - 60$

7) $25v^3 + 5v^2 + 30v + 6$

8) $105n^3 + 175n^2 - 75n - 125$

9) $96n^3 - 84n^2 + 112n - 98$

10) $28v^3 + 16v^2 - 21v - 12$

11) $4v^3 - 12v^2 - 5v + 15$

12) $49x^3 - 35x^2 + 56x - 40$

13) $24p^3 + 15p^2 - 56p - 35$

14) $24r^3 - 64r^2 - 21r + 56$

TOPIC 18: FACTORING TRINOMIALS ($a = 1$)

Factor each completely.

1) $b^2 + 8b + 7$

2) $n^2 - 11n + 10$

3) $m^2 + m - 90$

4) $n^2 + 4n - 12$

5) $n^2 - 10n + 9$

6) $b^2 + 16b + 64$

7) $m^2 + 2m - 24$

8) $x^2 - 4x + 24$

9) $k^2 - 13k + 40$

10) $a^2 + 11a + 18$

11) $n^2 - n - 56$

12) $n^2 - 5n + 6$

TOPIC 19: FACTORING TRINOMIALS ($a \neq 1$)

Factor each completely.

1) $3p^2 - 2p - 5$

2) $2n^2 + 3n - 9$

3) $3n^2 - 8n + 4$

4) $5n^2 + 19n + 12$

5) $2v^2 + 11v + 5$

6) $2n^2 + 5n + 2$

7) $7a^2 + 53a + 28$

8) $9k^2 + 66k + 21$

9) $15n^2 - 27n - 6$

10) $5x^2 - 18x + 9$

11) $4n^2 - 15n - 25$

12) $4x^2 - 35x + 49$

13) $4n^2 - 17n + 4$

14) $6x^2 + 7x - 49$

TOPIC 20: FACTORING SPECIAL CASES

Factor each completely.

1) $16n^2 - 9$

2) $4m^2 - 25$

3) $16b^2 - 40b + 25$

4) $4x^2 - 4x + 1$

5) $9x^2 - 1$

6) $n^2 - 25$

7) $n^4 - 100$

8) $a^4 - 9$

9) $k^4 - 36$

10) $n^4 - 49$

11) $98n^2 - 200$

12) $3 + 6b + 3b^2$

13) $400 - 36v^2$

14) $100x^2 + 180x + 81$

15) $10n^2 + 100n + 250$

16) $49n^2 - 56n + 16$

TOPIC 21: SOLVING QUADRATICS BY FACTORING

Solve each equation by factoring.

1) $(k + 1)(k - 5) = 0$

2) $(a + 1)(a + 2) = 0$

3) $(4k + 5)(k + 1) = 0$

4) $(2m + 3)(4m + 3) = 0$

5) $x^2 - 11x + 19 = -5$

6) $n^2 + 7n + 15 = 5$

7) $n^2 - 10n + 22 = -2$

8) $n^2 + 3n - 12 = 6$

9) $6n^2 - 18n - 18 = 6$

10) $7r^2 - 14r = -7$

11) $n^2 + 8n = -15$

12) $5r^2 - 44r + 120 = -30 + 11r$

TOPIC 22: SOLVING QUADRATICS BY COMPLETING THE SQUARE

Solve each equation by completing the square.

1) $p^2 + 14p - 38 = 0$

2) $v^2 + 6v - 59 = 0$

3) $a^2 + 14a - 51 = 0$

4) $x^2 - 12x + 11 = 0$

5) $x^2 + 6x + 8 = 0$

6) $n^2 - 2n - 3 = 0$

7) $x^2 + 14x - 15 = 0$

8) $k^2 - 12k + 23 = 0$

9) $r^2 - 4r - 91 = 7$

10) $x^2 - 10x + 26 = 8$

11) $k^2 - 4k + 1 = -5$

12) $b^2 + 2b = -20$

TOPIC 23: SOLVING QUADRATICS BY DISCRIMINANT

Solve each equation with the quadratic formula.

1) $m^2 - 5m - 14 = 0$

2) $b^2 - 4b + 4 = 0$

3) $2m^2 + 2m - 12 = 0$

4) $2x^2 - 3x - 5 = 0$

5) $x^2 + 4x + 3 = 0$

6) $2x^2 + 3x - 20 = 0$

7) $4b^2 + 8b + 7 = 4$

8) $2m^2 - 7m - 13 = -10$

TOPIC 24: SOLVING QUADRATICS BY TAKING SQUARE ROOTS

Solve each equation by taking square roots.

1) $k^2 = 76$

2) $k^2 = 16$

3) $x^2 = 21$

4) $a^2 = 4$

5) $x^2 + 8 = 28$

6) $2n^2 = -144$

7) $-6m^2 = -414$

8) $7x^2 = -21$

9) $m^2 + 7 = 88$

10) $-5x^2 = -500$

11) $-7n^2 = -448$

12) $-2k^2 = -162$

13) $x^2 - 5 = 73$

14) $16n^2 = 49$

TOPIC 25: PYTHAGOREAN THEOREM

Find the missing side of each right triangle. Give an approximate solution.

1) $a = 15$ mi, $c = 113$ mi

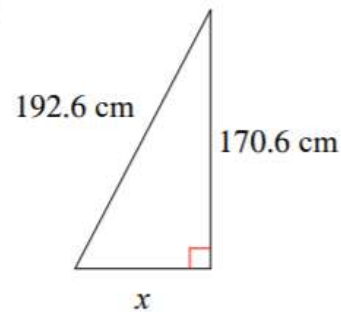
2) $a = 84$ km, $c = 159$ km

3) $a = 66$ yd, $b = 88$ yd

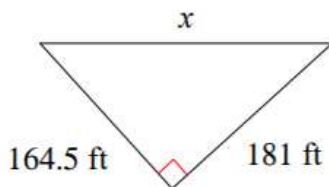
4) $a = 111$ km, $c = 185$ km

5) $a = 63$ yd, $c = 105$ yd

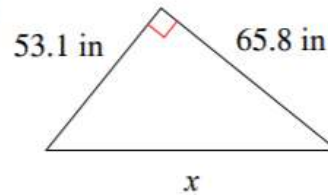
6)



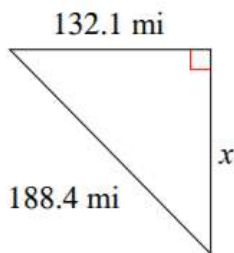
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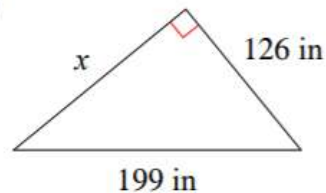
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9)



10)



Find the missing side of each right triangle. Give the exact solution.

11) $b = 14$ yd, $c = 15$ yd

12) $a = 13$ m, $c = 19$ m

13) $b = \sqrt{65}$ km, $c = 13$ km

14) $a = \sqrt{170}$ cm, $b = 13$ cm

15) $b = \sqrt{14}$ ft, $c = \sqrt{19}$ ft

16) $a = 8$ m, $b = 10$ m

TOPIC 26: DIRECT VARIATION (PROPORTIONAL)

Solve each proportion using Scaling or Cross Multiplication Method

$$1) \frac{m-12}{9} = \frac{m-5}{11}$$

$$2) \frac{x+11}{x-10} = \frac{7}{3}$$

$$3) \frac{p-9}{2} = \frac{p+1}{8}$$

$$4) \frac{12}{n+5} = \frac{8}{n-12}$$

$$5) \frac{2}{m+11} = -\frac{6}{m-7}$$

$$6) -\frac{7}{5} = \frac{b-7}{b-6}$$

$$7) \frac{k-4}{k-7} = \frac{11}{2}$$

$$8) \frac{a+5}{2} = \frac{a+11}{6}$$

$$9) \frac{m+4}{m-11} = -\frac{10}{6}$$

$$10) \frac{a-3}{7} = \frac{a+12}{4}$$

$$11) -\frac{7}{v+9} = -\frac{6}{v-9}$$

$$12) -\frac{10}{b-5} = -\frac{5}{b+7}$$

$$13) \frac{7}{4} = \frac{n-12}{n-4}$$

$$14) \frac{4}{a-11} = \frac{2}{a+2}$$

$$15) \frac{x-1}{8} = \frac{x+12}{5}$$

$$16) \frac{k+7}{k+3} = \frac{12}{7}$$

$$17) \frac{x-7}{x-8} = \frac{12}{4}$$

$$18) -\frac{5}{x+6} = \frac{3}{x+5}$$

$$19) -\frac{7}{10} = \frac{x-12}{x+8}$$

$$20) \frac{b+3}{5} = \frac{b-8}{6}$$

Solve each problem involving direct or inverse variation.

13) If x varies directly as y , and $x = 27$ when $y = 6$, find x when $y = 2$.

14) If y varies inversely as x , and $y = 23$ when $x = 8$, find y when $x = 4$.

15) If z varies directly as x , and $z = 30$ when $x = 8$, find z when $x = 4$.

16) If y varies inversely as x , and $y = 14$ when $x = 8$, find y when $x = 7$.

17) If d varies directly as t , and $d = 150$ when $t = 3$, find d when $t = 5$.

18) If y varies directly as x , and $y = 6$ when $x = 10$, find x when $y = 18$.

19) If x varies inversely as y , and $x = 3$ when $y = 8$, find y when $x = 4$.

20) If z varies inversely as x^2 , and $z = 9$ when $x = \frac{2}{3}$, find z when $x = \frac{5}{4}$.

21) If y varies directly as x , and $y = -4$ when $x = 32$, find y when $x = 3$.

22) If p varies inversely as q^2 , and $p = 4$ when $q = \frac{1}{2}$, find p when $q = \frac{3}{2}$.

Solve each problem.

- 23) The number of pencils sold varies directly as the cost. If 5 pencils cost \$0.45, find the cost of 7 pencils.
- 24) On a scale drawing, 2 feet represents 30 yards. How many yards are represented by 3 feet?
- 25) On a map, 180 miles are represented by 4 inches. How many miles are represented by 6 inches?
- 26) The bending of a beam varies directly as its mass. A beam is bent 20mm by a mass of 40 kg. How much will the beam bend with a mass of 100 kg?
- 27) Y varies directly as the square of x . If y is 25 when x is 3, find y when x is 2.
- 28) The distance needed to stop a car varies directly as the square of its speed. It requires 120 m to stop a car at 70 km/h. What distance is required to stop a car at 80 km/h?
- 29) Laura has a mass of 60 kg and is sitting 265 cm from the fulcrum of a seesaw. Bill has a mass of 50 kg. How far from the fulcrum must he be to balance the seesaw? (Hint: The distance from the fulcrum varies inversely as the mass).
- 30) Tina's mass is 40 kg, and she is sitting 2 m from the fulcrum of a seesaw. Jasmine's mass is 20 kg. How far from the fulcrum must she sit to balance the seesaw?
- 31) Time varies inversely as speed if the distance is constant. A trip takes 4 hours at 80 km/h. How long does it take at 64 km/h?
- 32) In an electric circuit, the current varies inversely as the resistance. The current is 40 amps when the resistance is 12 ohms. Find the current when the resistance is 20 ohms.
- 33) The number of hours required to do a job varies inversely as the number of people working. It takes 8 hours for 4 people to paint the inside of a house. How long would it take 5 people to do the job?
- 34) The length of the base of a triangle with constant area varies inversely as the height. When the base is 18 cm long, the height is 7 cm. Find the length of the base when the height is 6 cm.

TOPIC 27: USING MIDPOINT FORMULA

Let $A(x_1, y_1), B(x_2, y_2)$, if M is the midpoint of the segment AB then

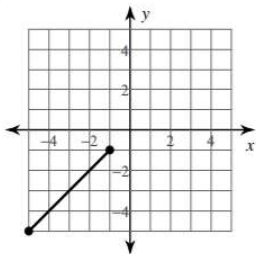
$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example: $A(1, 2), B(-2, 4)$. What is the midpoint coordinates of the segment AB?

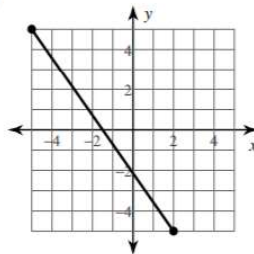
$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + (-2)}{2}, \frac{2 + 4}{2}\right) = \left(-\frac{1}{2}, 3\right)$$

Find the midpoint of each line segment.

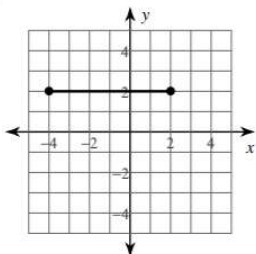
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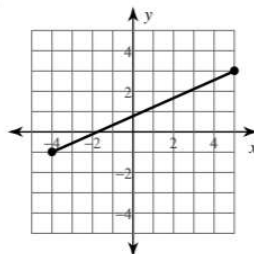
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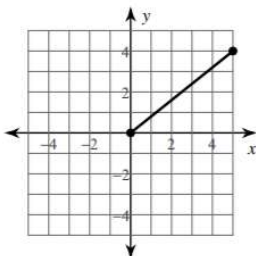
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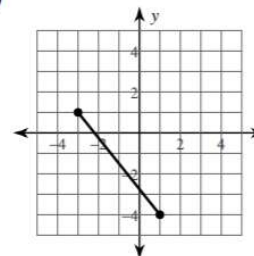
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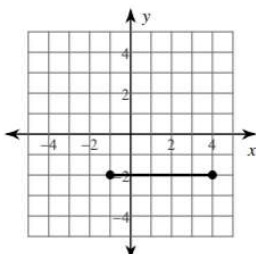
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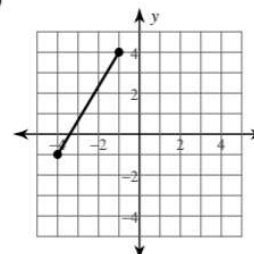
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7)



8)



TOPIC 28: USING DISTANCE FORMULA

Let $A(x_1, y_1), B(x_2, y_2)$, the distance between A and B is

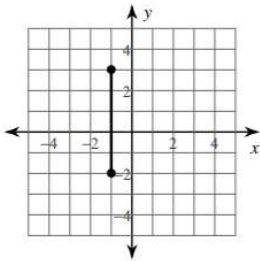
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: $A(1, 2), B(-2, 4)$. What is the distance between A and B?

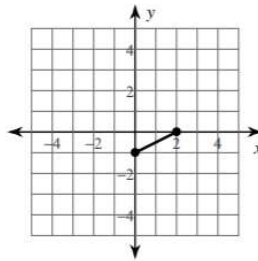
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2 - 1)^2 + (4 - 2)^2} = \sqrt{9 + 4} = \sqrt{13}$$

Find the distance between each pair of points.

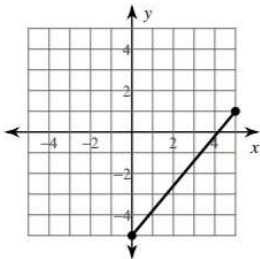
13)



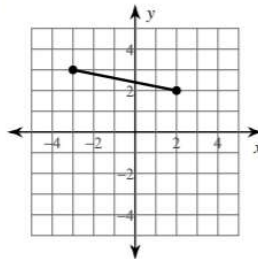
14)



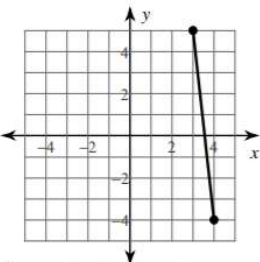
15)



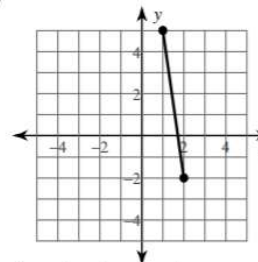
16)



17)



18)



19) $(0, -2), (-5, -1)$

20) $(6, 4), (-5, -1)$

21) $(3, 8), (9, 10)$

22) $(10, 1), (9, -4)$

23) $(-8, 10), (-6, 7)$

24) $(-5, 6), (8, -4)$

Critical thinking questions:

25) Name a point that is $\sqrt{2}$ away from $(-1, 5)$.

26) Name a point that is between 50 and 60 units away from $(7, -2)$ and state the distance between the two points.

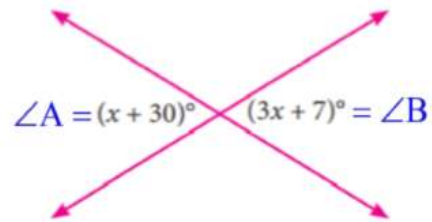
TOPIC 29: ANGLE RELATIONSHIPS

4. Solving for vertical angles

Example 3: Use the diagram at right to

a. Solve for x .

b. Find the measure of each angle.



Step 1: Observe that angle $(x+30)^\circ$ and angle $(3x+7)^\circ$ are **vertical angles** so they are **congruent (equal)**.

Step 2: Set up an equation and solve for x .

$$\begin{array}{r} x + 30 = 3x + 7 \\ -3x \quad -3x \\ \hline -2x + 30 = 7 \\ -30 \quad -30 \\ \hline -2x = -23 \\ -2 \quad -2 \\ \hline x = 11.5 \end{array}$$

Write the equation

Subtract $3x$ both sides

Simplify

Subtract 30 both sides

Simplify

Divide -2 both sides

Simplify

Step 3: Find the measure of each angle.

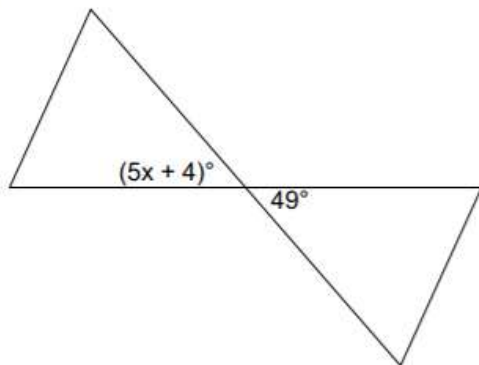
$$\angle A = x + 30 = 11.5 + 30 = 41.5^\circ$$

$$\angle B = 3x + 7 = 3(11.5) + 7 = 41.5^\circ$$

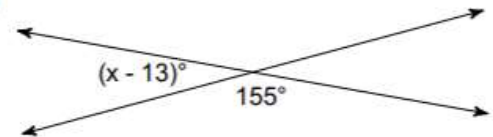
Check work:

$$41.5^\circ = 41.5^\circ \checkmark$$

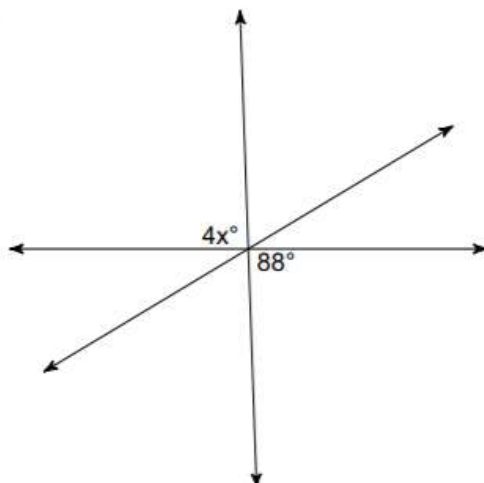
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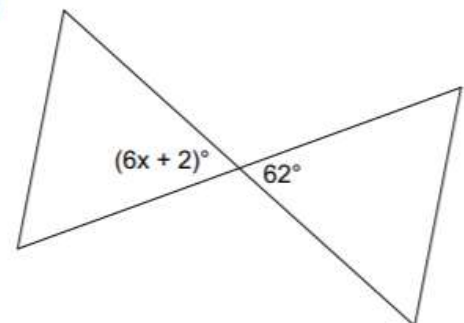
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5)



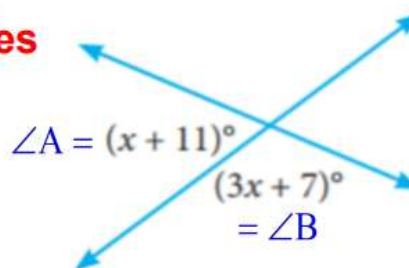
6)



6. Solving for supplementary angles

Example 5: Use the diagram at right to

- Solve for x .
- Find the measure of each angle.



Step 1: Observe that angle $(x+11)^\circ$ and angle $(3x+7)^\circ$ are **supplementary** angles so they add up to **180°** .

Step 2: Set up an equation and solve for x .

$x + 11 + 3x + 7 = 180$	Write the equation
$4x + 18 = 180$	Combine like terms
$\begin{array}{r} -18 \\ \hline 4x = 162 \end{array}$	Subtract 18 both sides
$x = 40.5$	Simplify
	Divide both sides by 4

Step 3: Find the measure of each angle.

$$\angle A = x + 11 = 40.5 + 11 = 51.5^\circ$$

$$\angle B = 3x + 7 = 3(40.5) + 7 = 128.5^\circ$$

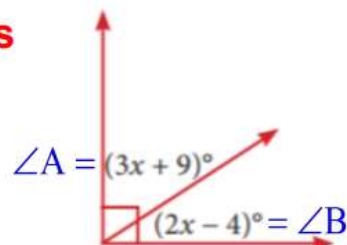
Check work:

$$51.5^\circ + 128.5^\circ = 180^\circ \quad \checkmark$$

8. Solving for complementary angles

Example 7: Use the diagram at right to

- Solve for x .
- Find the measure of each angle.



Step 1: Observe that angle $(3x+9)^\circ$ and angle $(2x-4)^\circ$ are **complementary** angles so they add up to **90°** .

Step 2: Set up an equation and solve for x .

$3x + 9 + 2x - 4 = 90$	Write the equation
$5x + 5 = 90$	Combine like terms
$\begin{array}{r} -5 \\ \hline 5x = 85 \end{array}$	Subtract 5 both sides
$x = 17$	Simplify
	Divide both sides by 5

Step 3: Find the measure of each angle.

$$\angle A = 3x + 9 = 3(17) + 9 = 60^\circ$$

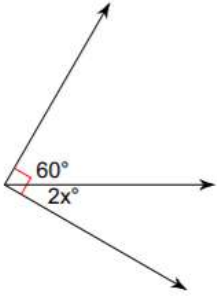
$$\angle B = 2x - 4 = 2(17) - 4 = 30^\circ$$

Check work:

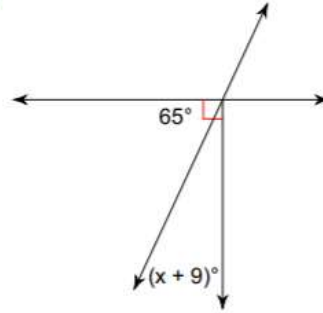
$$60^\circ + 30^\circ = 90^\circ \quad \checkmark$$

Find the value of x .

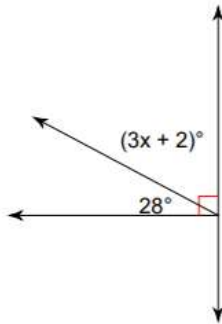
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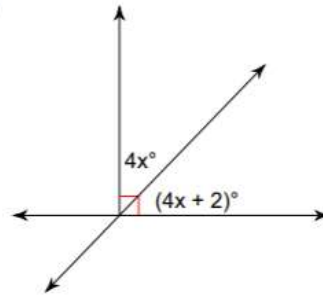
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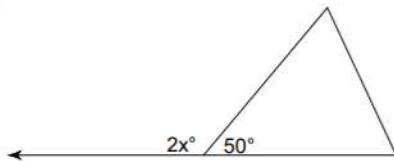
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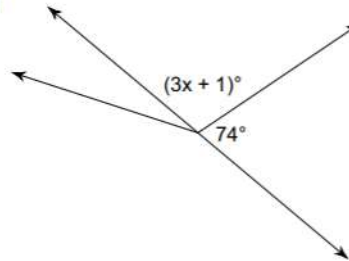
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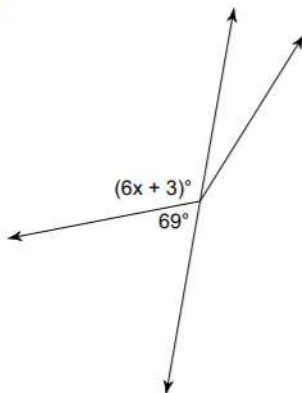
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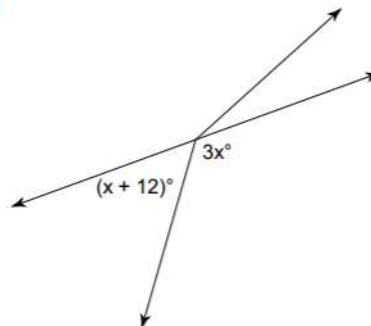
6)



7)



8)



TOPIC 30: CIRCUMFERENCE & AREA OF CIRCLES

2. Circumference of a circle (called C)

To find circumference (C) of a circle, we use either two following formulas depending on the context:

If radius is given, use this formula

$$C = 2\pi r$$

To find radius, use this formula

$$r = \frac{C}{2\pi}$$

If diameter is given, use this formula

$$C = \pi d$$

To find diameter, use this formula

$$d = \frac{C}{\pi}$$

Note: If the problem asks you to find the circumference of the circle in terms of π , it means you must keep π in the answer instead of using 3.14.

6. Area of a circle (called A)

To find area (A) of a circle, we use the following formula:

If radius is given, use this formula

$$r^2 = r \times r$$

If diameter is given, use this formula to calculate the radius first, then apply the formula (*) to calculate area.

Or If diameter is given, you can use this direct formula to calculate the area.

$$A = \pi r^2 \quad (*)$$

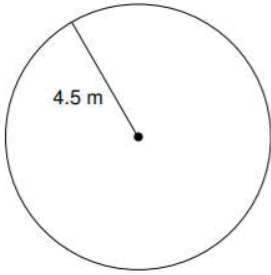
$$r = \frac{d}{2}$$

$$A = \pi \left(\frac{d}{2} \right)^2$$

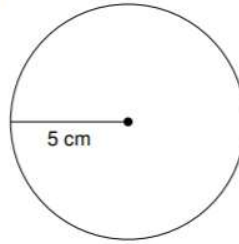
Note: If the problem asks you to find the circumference of the circle in terms of π , it means you must keep π in the answer instead of using 3.14.

Find the circumference of each circle. Round to the nearest tenth.

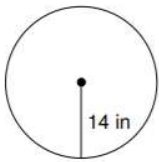
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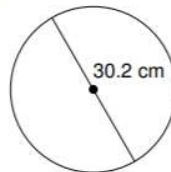
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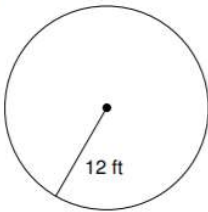


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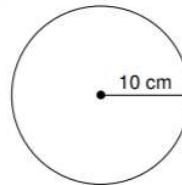


Find the area of each. Round to the nearest tenth.

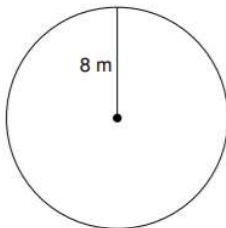
9)



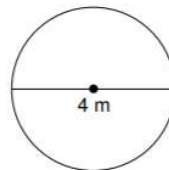
10)



11)



12)



Find the diameter of each circle.

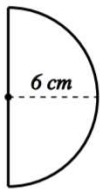
17) area = 4π in²

18) area = 49π yd²

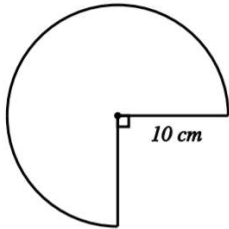
19) circumference = 162π yd

20) circumference = 30π yd

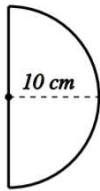
1) Find the area of the semicircle, rounding your answer to 3 significant figures



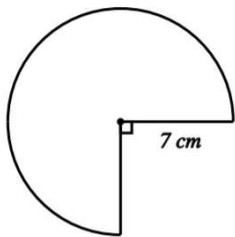
2) Find the area of the three-quarter circle, rounding your answer to 3 significant figures



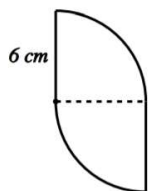
7) Find the perimeter of the semicircle, rounding your answer to 3 significant figures



9) Find the perimeter of the three-quarter circle, rounding your answer to 3 significant figures

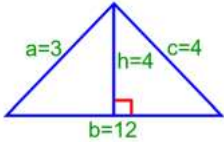

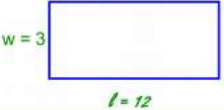
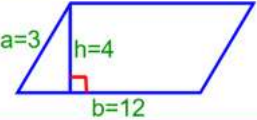
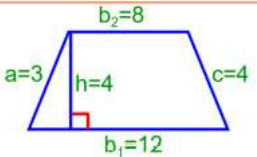


10) Find the perimeter of the shape below, rounding your answer to 3 significant figures



TOPIC 31: PERIMETER & AREA OF COMPLEX FIGURES

2. Area of simple figures

Figure	Name	Perimeter	Area	Example
	Triangle	$P = a + b + c$ $= 3 + 12 + 4$ $= 19$	$A = \frac{bh}{2}$	$A = \frac{bh}{2} = \frac{12(4)}{2} = 24$
	Square	$P = s(4)$ $= 4(4) = 16$	$A = s^2$	$A = s^2 = 4^2 = 16$
	Rectangle	$P = 2(l + w)$ $= 2(12 + 3) = 30$	$A = lw$	$A = lw = 12(3) = 36$
	Parallelogram	$P = 2(a + b)$ $= 2(3 + 12) = 30$	$A = bh$	$A = bh = 12(4) = 48$
	Trapezoid	$P = a + b_1 + c + b_2$ $= 3 + 12 + 4 + 8$ $= 27$	$A = \frac{(b_1 + b_2)h}{2}$	$A = \frac{(b_1 + b_2)h}{2} = \frac{(12 + 8)4}{2} = 40$

Method 1: Break composite figures down

Step 1: Break composite figures down into simple shapes

Step 2: Calculate area of each simple shape.

Step 3: Add up all areas from steps 2.

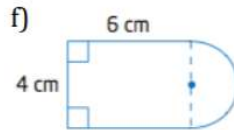
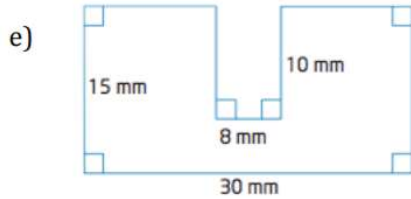
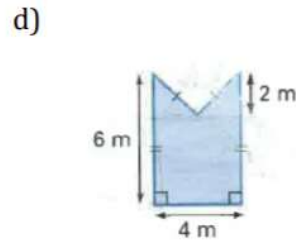
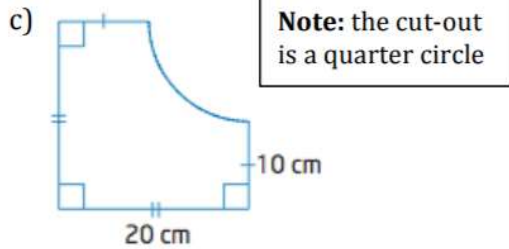
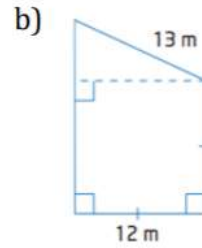
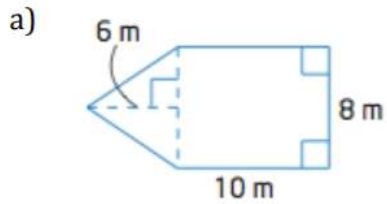
Method 2: Create new composite figures

Step 1: Create a shape **larger than** the composite figure then find its area.

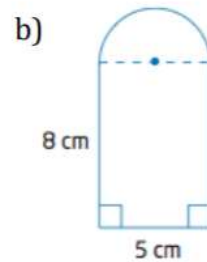
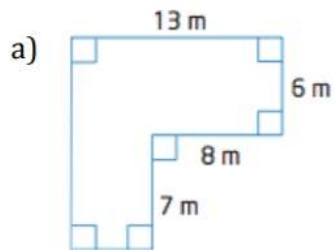
Step 2: Calculate the area of the pieces **not included** in the composite shape.

Step 3: Area of composite figure = Area (step 1) - Area (step 2).

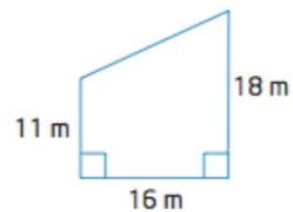
1. Find the area of each of the composite figures:



2. Find the perimeter of each of the composite figures:



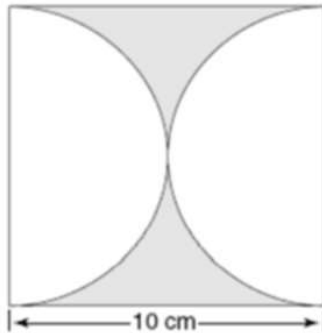
3. a) What length of fencing is needed to surround this yard?



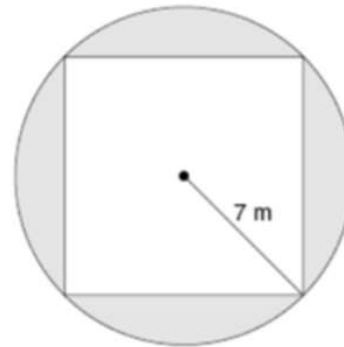
b) What is the area of the yard?

Find the area of the shaded region in each of the following figures.

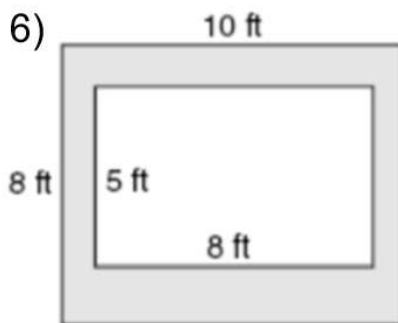
4)



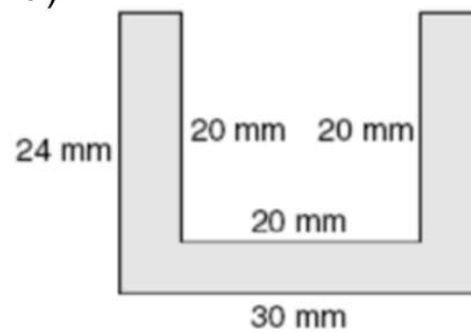
5)



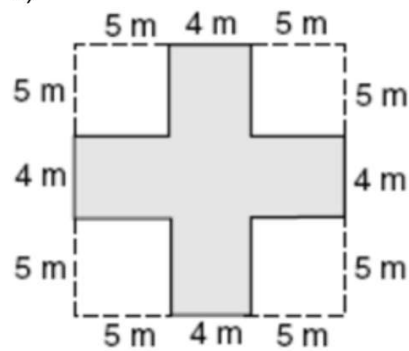
6)



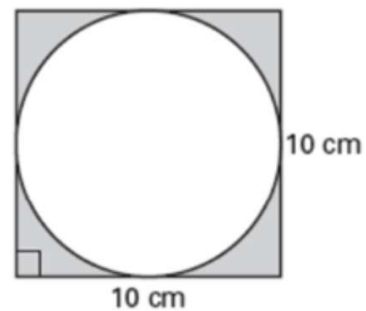
7)



8)

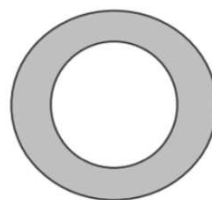


9)

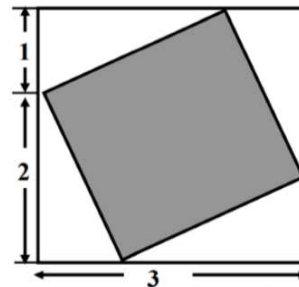


10)

This figure consists of 2 concentric circles. If the shaded area is 64π sq. in. and the smaller circle has a radius of 6 in., what is the radius, in inches of the larger circle?

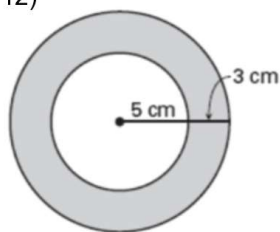


11)

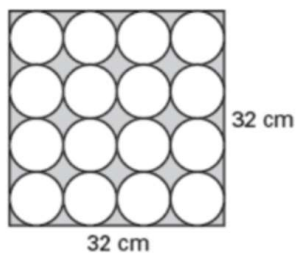


The shaded square is inscribed in the larger square

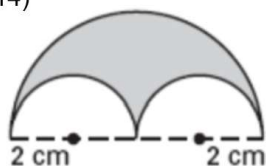
12)



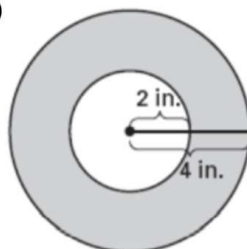
13)



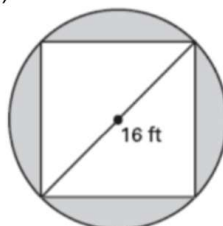
14)



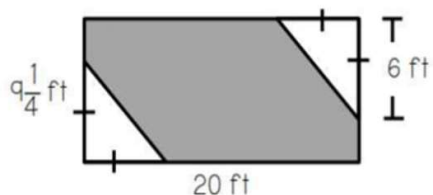
15)



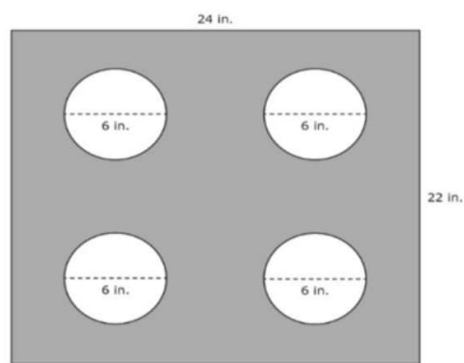
16)



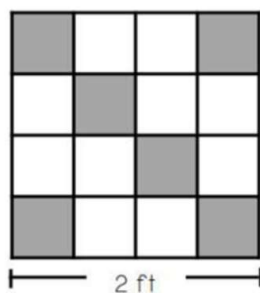
17) A garden is sodded in the shaded portion below. How many square feet were covered with sod?



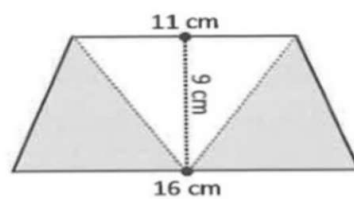
18)



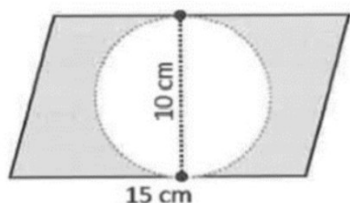
19) A square is divided into smaller squares and portions are shaded. What is the area of the shaded portion?



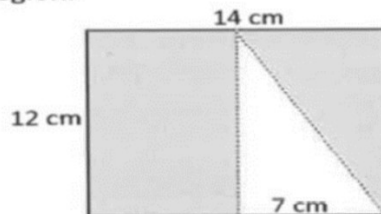
20) Find the area of the shaded region.



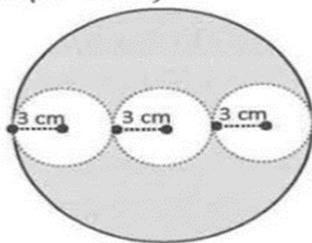
21) Find the area of the shaded region. ($\pi = 3.14$)



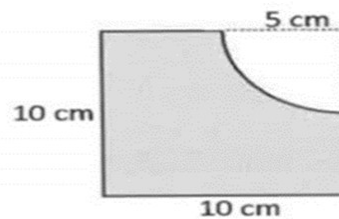
22) Find the area of the shaded region.



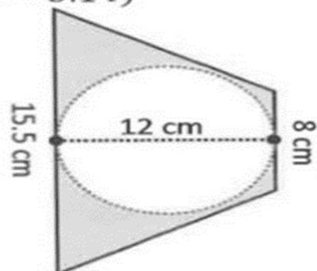
- 23) Find the area of the shaded region. ($\pi = 3.14$)



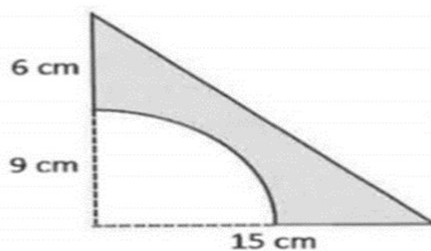
- 24) Find the area of the shaded region. ($\pi = 3.14$)



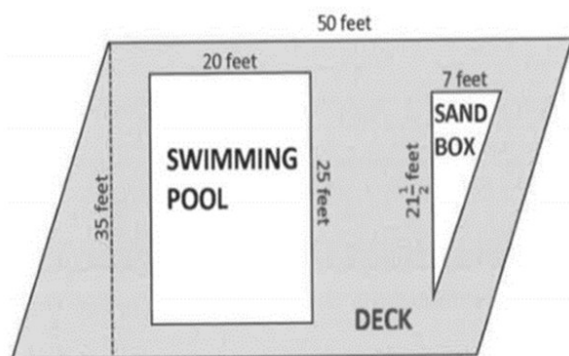
- 25) Find the area of the shaded region. ($\pi = 3.14$)



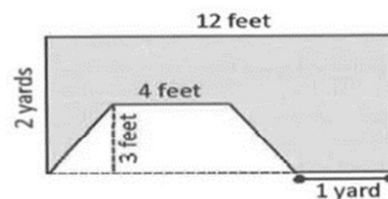
- 26) A triangular piece of wood was cut as shown below. Determine the area of the shaded region to the nearest square centimeter.



- 27) Mr. Clarke built a deck around the swimming pool and sandbox in his backyard. What is the area of the decking that surrounds the pool and sandbox, rounded to the nearest tenth of a square foot?

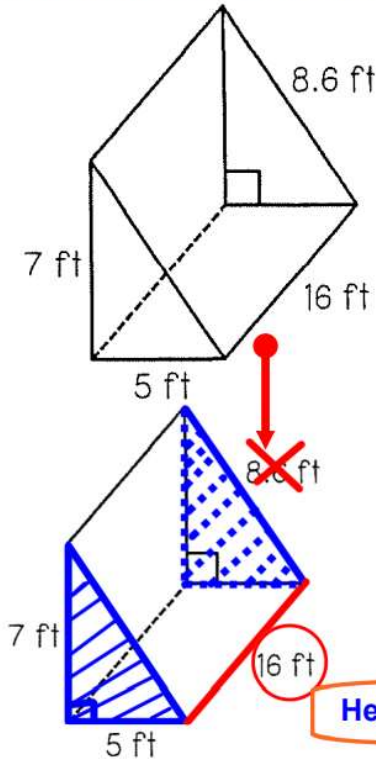


- 28) Welder Willis cut a shape from a sheet of metal. Determine the area of the remaining metal, measured in square feet.



TOPIC 32: VOLUME OF PRISM & PYRAMID

Example 2: Calculate the volume of the triangular prism.



Step 1: Identify 2 bases, H, h, b and cross out unnecessary information.

- Two bases are in blue lines
- H (red line) is 16 ft
- h is 7 ft, b is 5 ft; h and b make a right angle.
- Cross out 8.6 ft

Step 2: Calculate area of one base B by using the formula

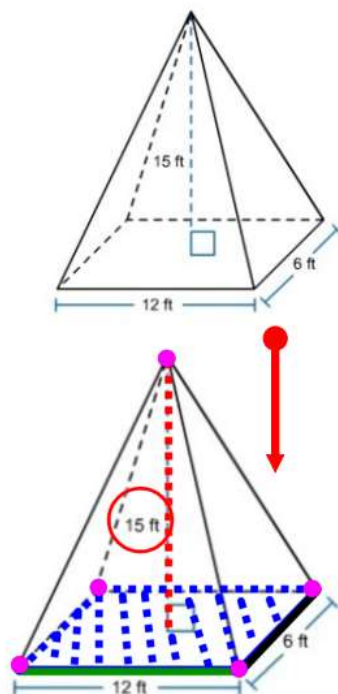
$$B = \frac{bh}{2} = \frac{5 \times 7}{2} = 17.5 \text{ ft}^2$$

Step 3: Calculate Volume V by using the formula: $V = B \times H$

$$V = BH = 17.5 \times 16 = 280 \text{ ft}^3$$

Hence the volume of the triangular prism is 280 ft³.

Example 2: Calculate the volume of rectangular pyramid



Step 1: Identify the Base, H, and l, w if the base is rectangle and cross out unnecessary information.

- The base is a rectangle in blue lines
- H (red dotted line) is 15ft
- l (green line) is 12ft, w (black line) is 6ft; l and w make a right angle.
- We have nothing to cross out

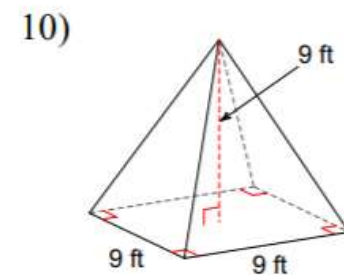
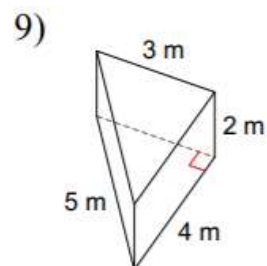
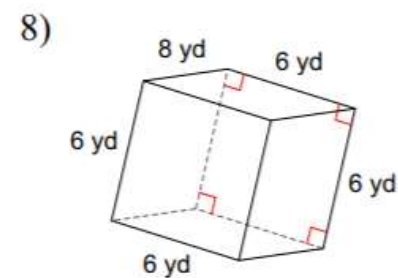
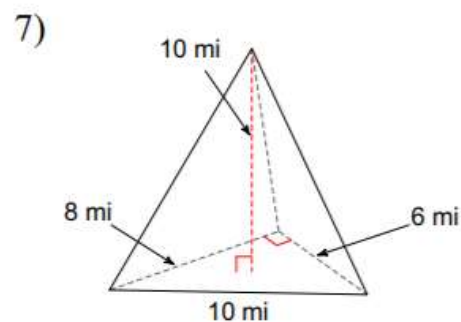
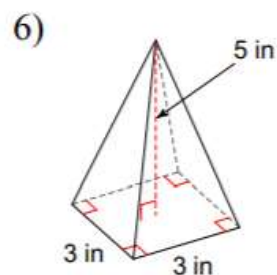
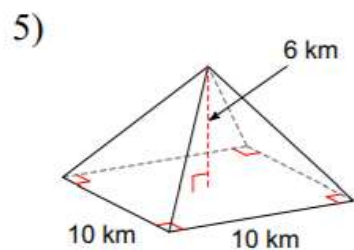
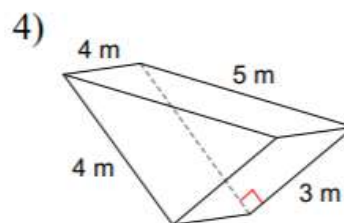
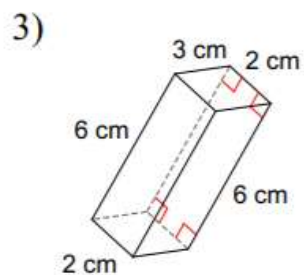
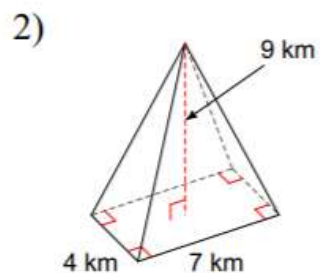
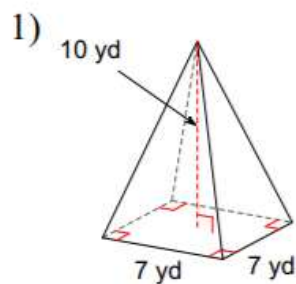
Step 2: Calculate area of the base B

$$B = lw = 12(6) = 72 \text{ ft}^2$$

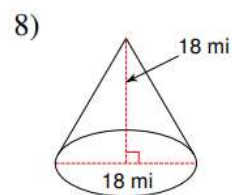
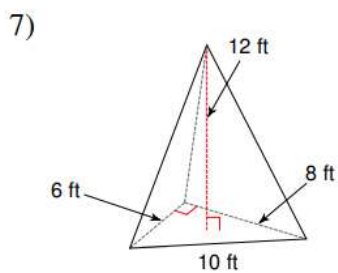
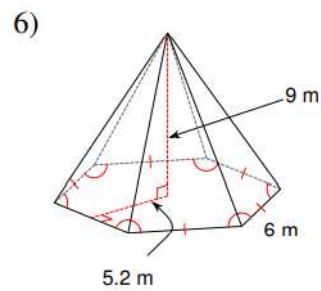
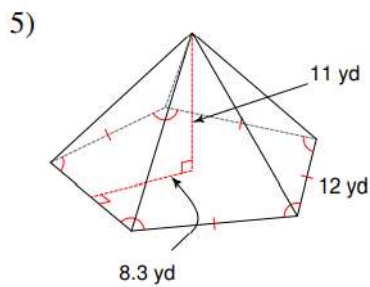
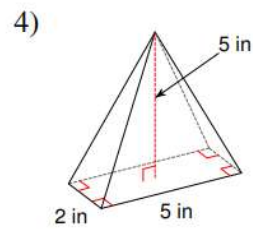
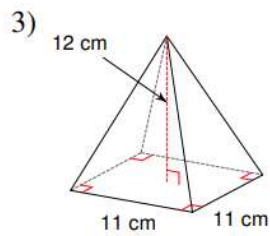
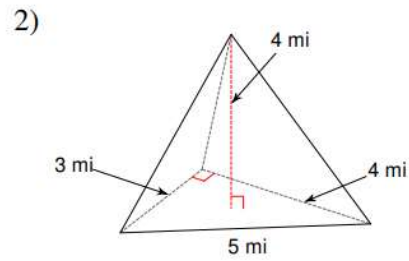
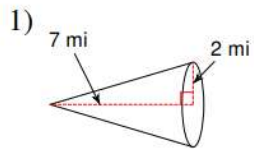
Step 3: Calculate Volume V

$$V = \frac{BH}{3} = \frac{72(15)}{3} = 360 \text{ ft}^3$$

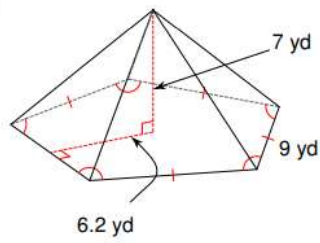
Find the volume of each figure. Round your answers to the nearest tenth, if necessary.



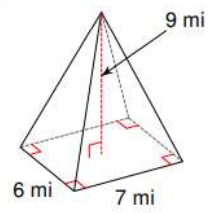
Find the volume of each figure. Round your answers to the nearest tenth, if necessary.



9)



10)



11) A square pyramid measuring 10 yd along each edge of the base with a height of 6 yd.

12) A pyramid 5 m tall with a right triangle for a base with side lengths 6 m, 8 m, and 10 m.

13) A cone with radius 4 m and a height of 12 m.

14) A hexagonal pyramid 11 ft tall with a regular base measuring 6 ft on each side and an apothem of length 5.2 ft.

TOPIC 33: PROPORTIONS AND SIMILAR FIGURES

4. Find Missing Side Lengths in Similar Figures

Step 1: Write corresponding angles in similar form.

Step 2: Set up a proportion.

Step 3: Apply Scaling Method or Cross Multiplication to solve for missing lengths.

Example 7: The perimeter of triangle GHI is 70 cm. If both triangles are similar, then what is the length of JL?

Step 1: Find the length GI

$$GI = 70 - (25 + 25) = 20 \text{ cm}$$

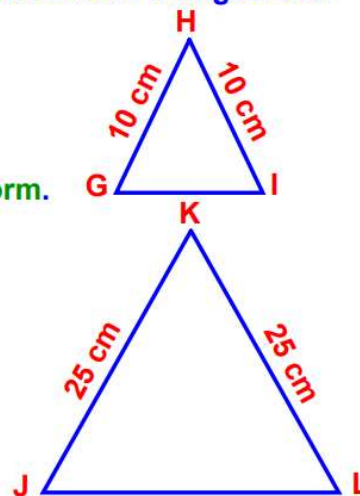
Step 2: Write corresponding angles in similar form.

$$\triangle GHI \sim \triangle JKL$$

Step 3: Set up a proportion and solve for JL.

$$\frac{GI}{JL} = \frac{GH}{JK}$$

$$\frac{20 \times 2}{JL \times 25} = \frac{10}{25} \Rightarrow JL = 25 \times 2 = 50 \text{ cm}$$



Example 10: (STAAR 7th Math 2016)

2 Figure JKLM is similar to figure PQRS.

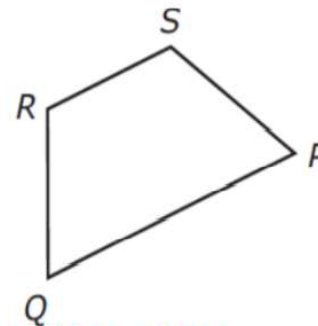
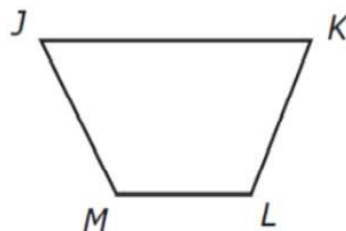
Which proportion must be true for these figures?

F $\frac{QR}{QP} = \frac{JK}{LM}$

G $\frac{QR}{KL} = \frac{RS}{JK}$

H $\frac{QR}{MJ} = \frac{PQ}{LM}$

J $\frac{QR}{KL} = \frac{PS}{JM}$



Step 1: Write corresponding angles in similar form: PQRS ~ JKLM

Step 2: Set up a proportion based on similar form in step 1.

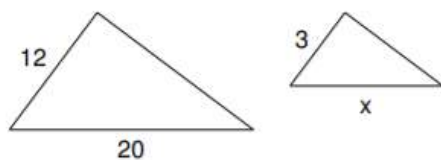
$$\frac{PQ}{JK} = \frac{PR}{JL} = \frac{PS}{JM} = \frac{QR}{KL} = \frac{QS}{KM} = \frac{RS}{LM} \quad (*)$$

Step 3: Compare with answer choices, pick the correct proportion in (*).

The correct answer is boxed in (*), hence the answer is J.

Each pair of figures is similar. Find the missing side.

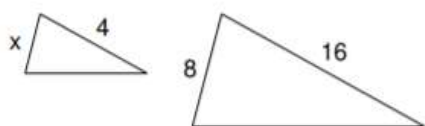
1)



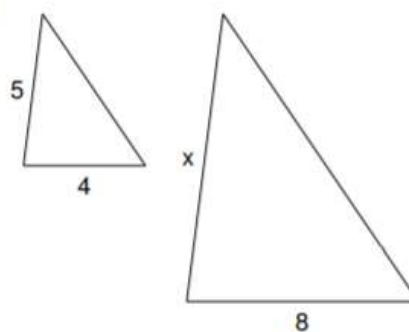
2)



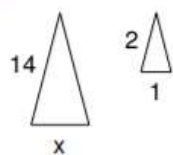
3)



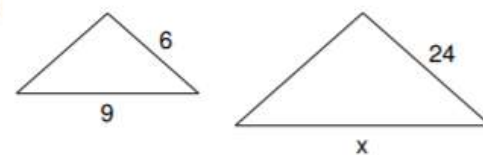
4)



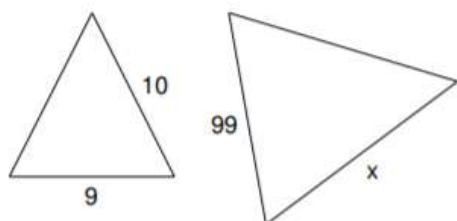
5)



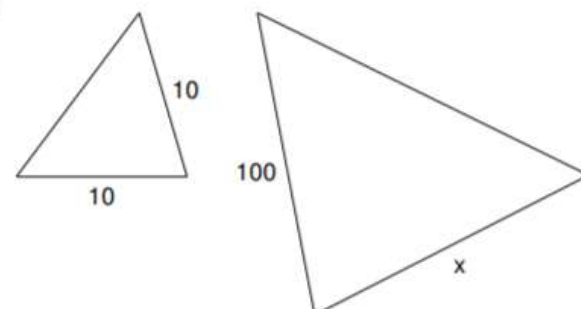
6)



7)

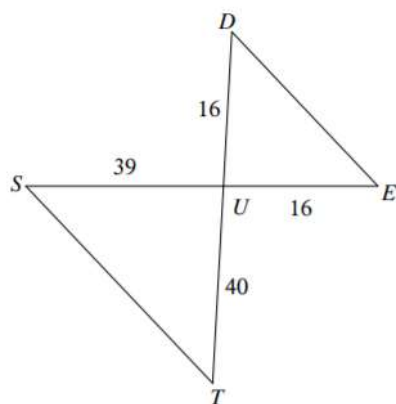


8)



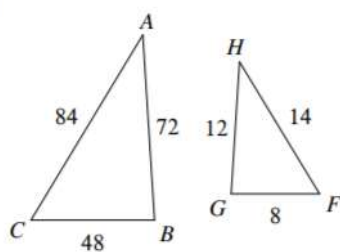
State if the triangles in each pair are similar. If so, state how you know they are similar and complete the similarity statement.

1)



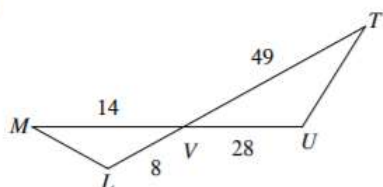
$\Delta UTS \sim$ _____

2)



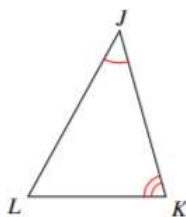
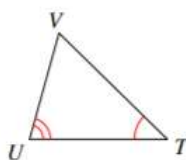
$\Delta CBA \sim$ _____

3)



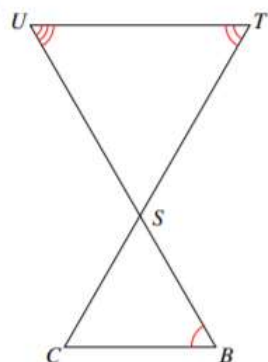
$\Delta VUT \sim$ _____

4)



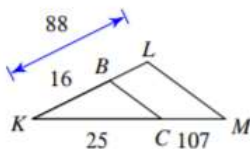
$\Delta JKL \sim$ _____

5)



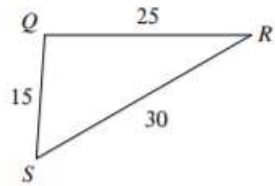
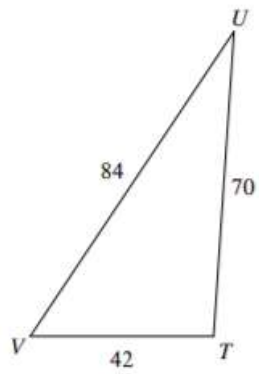
$\Delta STU \sim$ _____

6)



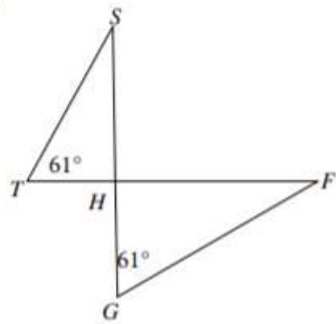
$\Delta KLM \sim$ _____

7)



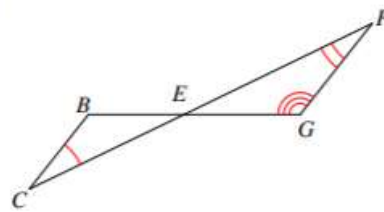
$\triangle TUV \sim \underline{\hspace{2cm}}$

9)



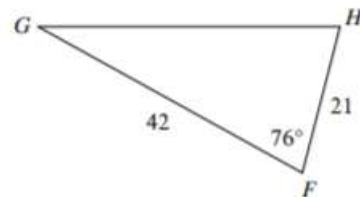
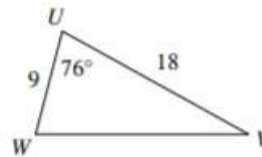
$\triangle HGF \sim \underline{\hspace{2cm}}$

8)



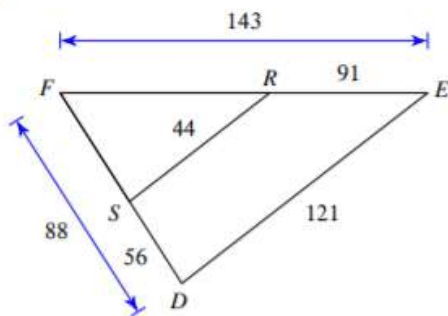
$\triangle EFG \sim \underline{\hspace{2cm}}$

10)



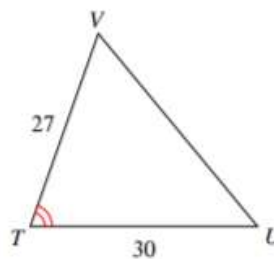
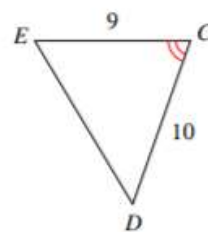
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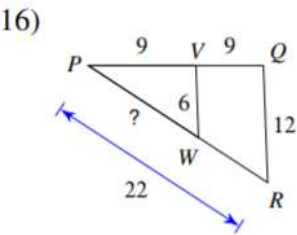
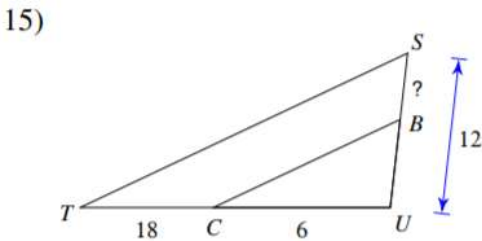
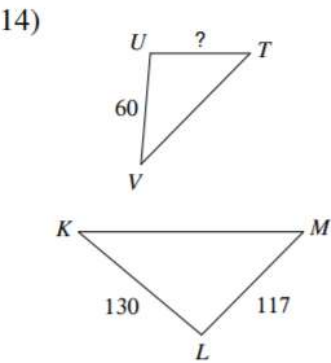
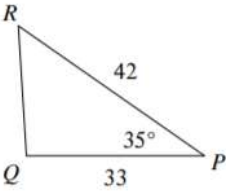
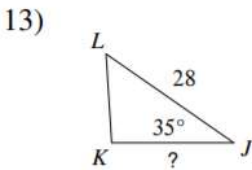
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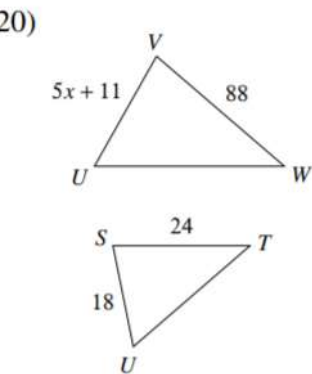
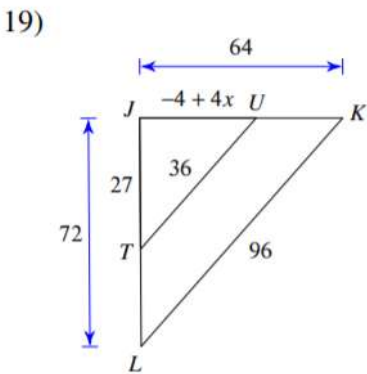
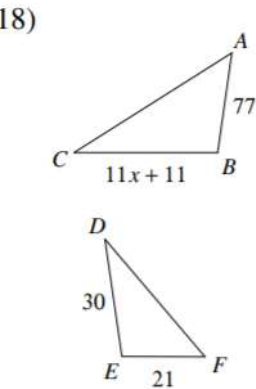
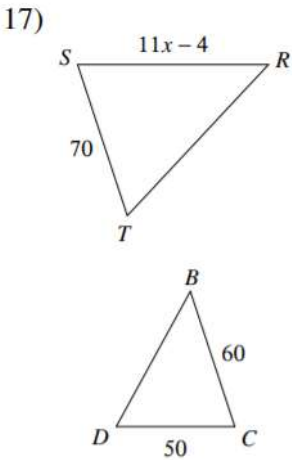


$\triangle TUV \sim \underline{\hspace{2cm}}$

Find the missing length. The triangles in each pair are similar.



Solve for x . The triangles in each pair are similar.



TOPIC 34: DEPENDENT - INDEPENDENT PROBABILITY

Dependent and Independent Probability

1. Independent Probability (Probability with Replacement):

- The outcome of 1 event **doesn't impact** the outcome of the other event.
- Independent Probability means that when you **pick something out**, you have to **put it back**; so the **total possible outcomes remain unchanged**.
- **Keywords:** replace, repeat, return it, put it back, replace it, 2 different bag, 2 coins, 2 cubes, 1 coin-1 cube, independent, does not impact.
- Do **Multiplication** if you see "and, with"; do **Addition** if you see "or".

$$P(A \text{ and } B) = P(A) \times P(B) = \frac{\# \text{ of } A}{\text{Total possible outcomes}} \times \frac{\# \text{ of } B}{\text{Total possible outcomes}}$$

Example 1

Fazio is selecting a jersey. His choices are shown below.



Fazio chooses a jersey at random and then **replaces it**. He then selects a second jersey at random. What is the probability that Fazio selects a striped jersey both times?

$$\begin{aligned} P(\text{striped jersey}) &= P(1\text{st one}) \times P(2\text{nd one}) \\ &= \frac{2}{10} \times \frac{2}{10} = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25} \end{aligned}$$

2. Dependent Probability (Probability without Replacement):

- The outcome of one event **impacts** the outcome of the other event.
- Dependent Probability means that when you **pick something out**, you **don't put it back**; therefore the **new total possible outcomes decrease**.
- **Keywords:** Without Replacement, no repetition, don't put it back, don't replace, don't return, leave it out, one choice after another.

$$P(A \text{ and } B) = P(A) \times P(B) = \frac{\# \text{ of } A \text{ options}}{\text{Total possible outcomes}} \times \frac{\# \text{ of } B \text{ options after } A}{\text{Total possible outcomes after } A}$$

Example 2

Fazio is selecting a jersey. His choices are shown below.



Fazio chooses a jersey at random and **doesn't replace it**. He then selects a second jersey at random. What is the probability that Fazio selects a striped jersey both times?

$$\begin{aligned} P(\text{striped jersey}) &= P(1\text{st one}) \times P(2\text{nd one}) \\ &= \frac{2}{10} \times \frac{(2-1)}{(10-1)} \\ &= \frac{2}{10} \times \frac{1}{9} = \frac{2}{90} = \frac{1}{45} \end{aligned}$$

Two cards are drawn from a standard deck of 52 cards one after another.

Problems

Work Space

Find the probability of drawing a king card on the first draw.

Answer: _____

If the first card is king and the card is not replaced, what is the probability of selecting a king on the second draw?

Answer: _____

Find the probability of selecting a king from the first draw and a queen on the second draw without replacing the first card.

Answer: _____

Find the probability of selecting a Jack on the first draw and a queen on the second draw after replacing the first card.

Answer: _____

Find the probability of selecting a 6 or 7 on the first draw and an 8 or 9 on the second draw without replacement.

Answer: _____

Determine whether the scenario involves independent or dependent events.

- 1) You flip a coin and then roll a fair six-sided die. The coin lands heads-up and the die shows a one.
- 2) A bag contains eight red marbles and four blue marbles. You randomly pick a marble and then pick a second marble without returning the marbles to the bag. The first marble is red and the second marble is blue.
- 3) A box of chocolates contains five milk chocolates, five dark chocolates, and five white chocolates. You randomly select and eat three chocolates. The first piece is milk chocolate, the second is dark chocolate, and the third is white chocolate.
- 4) A cooler contains ten bottles of sports drink: four lemon-lime flavored, three orange flavored, and three fruit-punch flavored. Three times, you randomly grab a bottle, return the bottle to the cooler, and then mix up the bottles. The first time, you get a lemon-lime drink. The second and third times, you get fruit-punch.

Find the probability.

- 5) You flip a coin and then roll a fair six-sided die. The coin lands heads-up and the die shows an even number.
- 6) You roll a fair six-sided die twice. The first roll shows a five and the second roll shows a six.
- 7) There are eight shirts in your closet, four blue and four green. You randomly select one to wear on Monday and then a different one on Tuesday. You wear blue shirts both days.
- 8) A basket contains five apples and seven peaches. You randomly select one piece of fruit and eat it. Then you randomly select another piece of fruit. The first piece of fruit is an apple and the second piece is a peach.

TOPIC 35: PROPERTIES OF EXPONENTS

Formula	Example	Formula	Example
$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^2 = 2^{3+2} = 2^5 = 32$	$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$	$\frac{1}{a^{-n}} = a^n$	$\frac{1}{2^{-3}} = 2^3 = 8$
$(a^m)^n = a^{m \cdot n}$	$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$	$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$
$(ab)^n = a^n \cdot b^n$	$(2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$	$a^{\frac{1}{2}} = \sqrt{a}$	$4^{\frac{1}{2}} = \sqrt{4} = 2$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$	$a^{\frac{1}{3}} = \sqrt[3]{a}$	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
$a^0 = 1$	$(-3)^0 = 1$	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$9^{\frac{2}{3}} = \sqrt[3]{9^2} = \sqrt[3]{81}$

Simplify. Your answer should contain only positive exponents.

1) $2m^2 \cdot 2m^3$

2) $m^4 \cdot 2m^{-3}$

3) $4r^{-3} \cdot 2r^2$

4) $4n^4 \cdot 2n^{-3}$

5) $2k^4 \cdot 4k$

6) $2x^3y^{-3} \cdot 2x^{-1}y^3$

7) $2y^2 \cdot 3x$

8) $4v^3 \cdot vu^2$

$$9) 4a^3b^2 \cdot 3a^{-4}b^{-3}$$

$$10) x^2y^{-4} \cdot x^3y^2$$

$$11) (x^2)^0$$

$$12) (2x^2)^{-4}$$

$$13) (4r^0)^4$$

$$14) (4a^3)^2$$

$$15) (3k^4)^4$$

$$16) (4xy)^{-1}$$

$$23) \frac{3n^4}{3n^3}$$

$$24) \frac{m^4}{2m^4}$$

$$25) \frac{3m^{-4}}{m^3}$$

$$26) \frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4}$$

$$27) \frac{4x^0y^{-2}z^3}{4x}$$

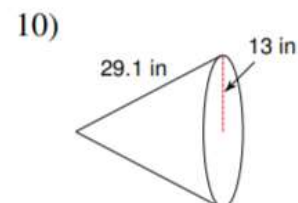
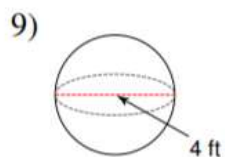
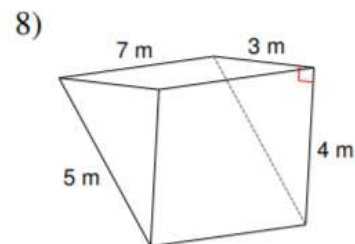
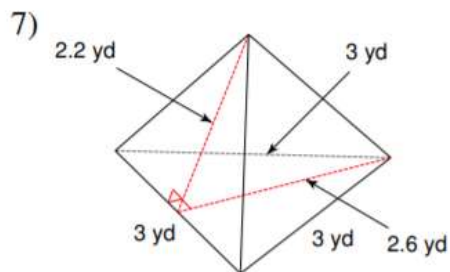
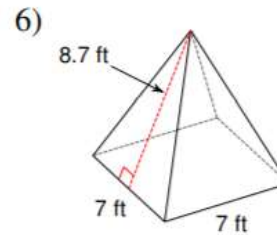
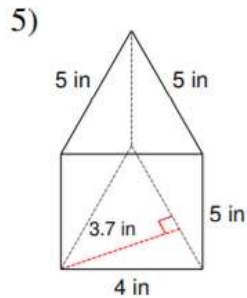
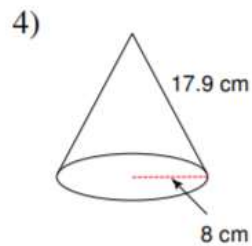
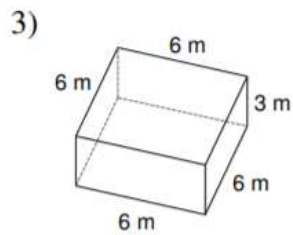
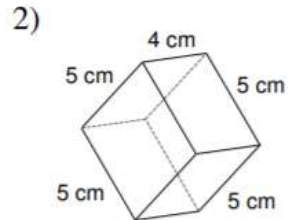
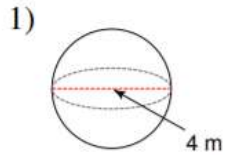
$$28) \frac{2h^3j^{-3}k^4}{3jk}$$

$$29) \frac{4m^4n^3p^3}{3m^2n^2p^4}$$

$$30) \frac{3x^3y^{-1}z^{-1}}{x^{-4}y^0z^0}$$

TOPIC 36: SURFACE AREA OF SOLIDS

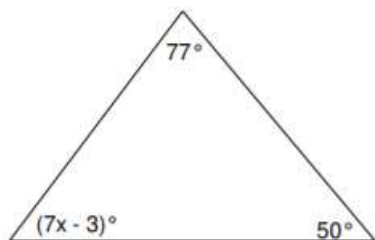
Find the surface area of each figure. Round to the nearest tenth.



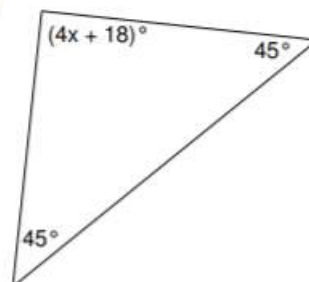
TOPIC 37: ANGLE SUM OF TRIANGLES&QUADRILATERALS

Find the value of x.

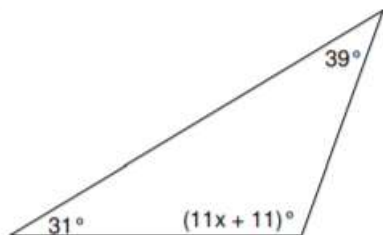
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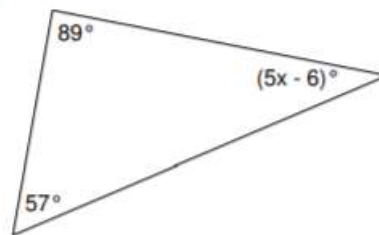
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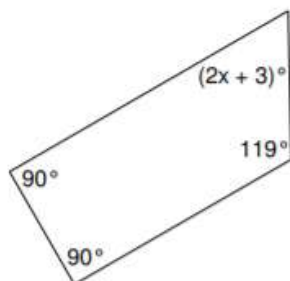
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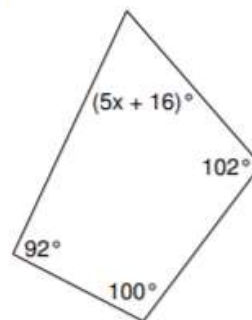
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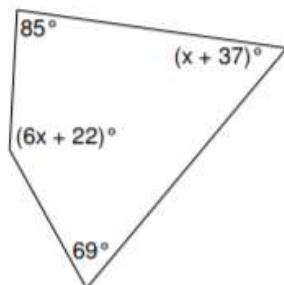
17)



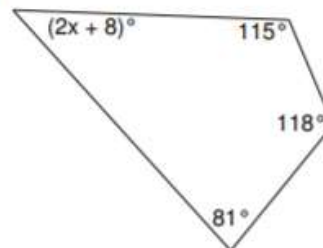
18)



19)



20)



TOPIC 38: TRANSFORMATION OF LINEAR FUNCTIONS

If $f(x) = x$, describe the transformations of $f(x)$ to get $g(x)$.

1) $g(x) = -(x + 5)$

2) $g(x) = \frac{2}{3}x - 4$

3) $g(x) = (x - 2) + 3$

4) $g(x) = 2(x + 1) - 10$

5) $g(x) = (x + 10) + 4$

6) $g(x) = -x + 15$

7) $g(x) = \frac{1}{2}(x - 5)$

8) $g(x) = -4(x - 2) - 4$

U2T3: I can transform linear functions.

Let $g(x)$ be the indicated transformation of $f(x) = x$. Write a rule for $g(x)$.

9) Translate up 3 units

10) Translate 5 units down

11) Reflect across the x-axis

12) Vertical stretch by a factor of 4

13) Vertical compression by a factor of $\frac{2}{3}$

14) Vertical compression by a factor of $\frac{1}{2}$
and a translation 4 units up

15) Reflect across the x-axis and translate 3
units left

16) Vertical stretch by a factor of 5 and
translated 6 units down

17) Vertical compression by a factor of $\frac{1}{3}$,
translate 1 unit right and 10 units up

18) Vertical stretch by a factor of 2, translated
7 left and 4 up, and reflected across the
x-axis

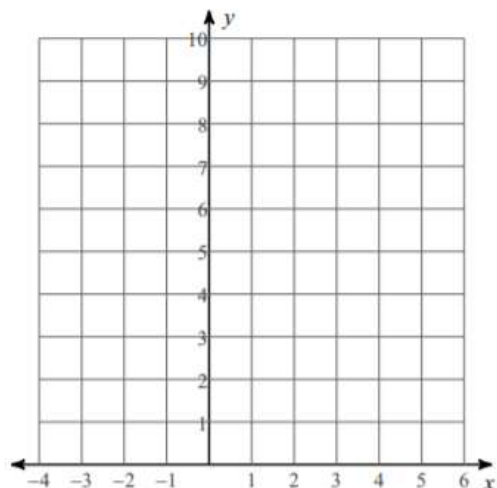
19) Translate 3 units down and 4 units right

20) Reflect across the x-axis, translate 2 down
and 1 left

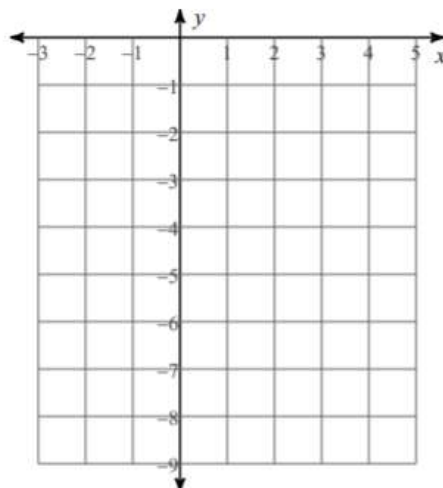
TOPIC 39: TRANSFORMATION OF QUADRATIC FUNCTIONS

List all transformations to the parent function indicated by each and then graph the function. State the Domain and Range.

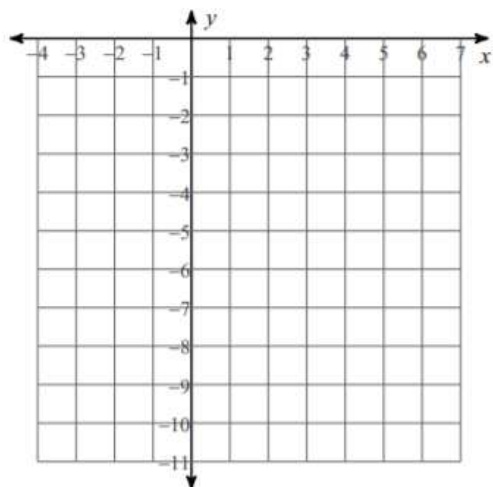
1) $y = 2(x - 1)^2 + 1$



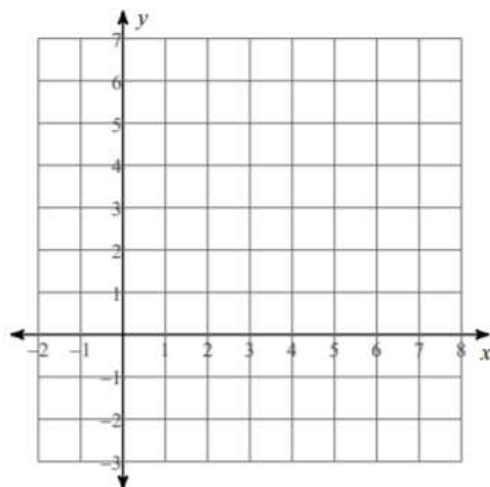
2) $y = -(x - 3)^2 - 4$



3) $y = -2(x - 4)^2 - 2$

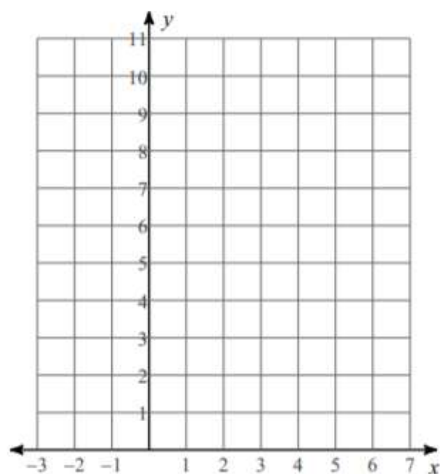


4) $y = 2(x - 3)^2 - 2$

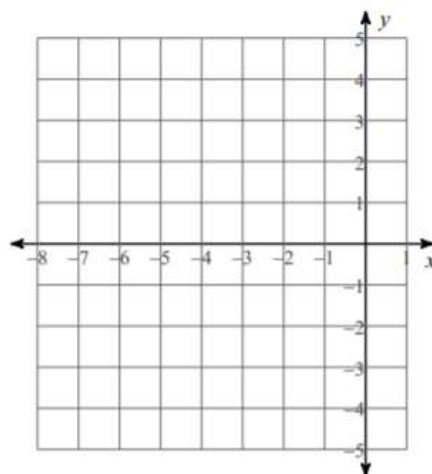


Convert to vertex form and then list all indicated transformations. Graph the function and state the domain and range.

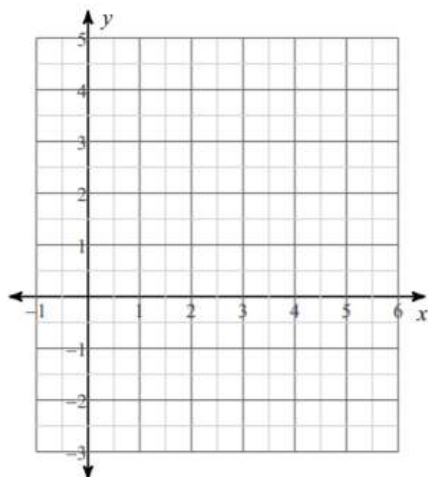
11) $y = 2x^2 - 4x + 4$



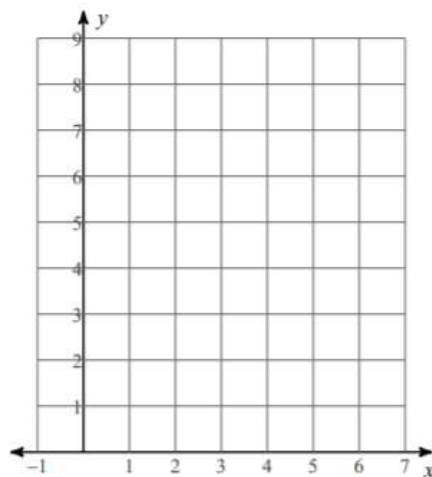
12) $y = -2x^2 - 16x - 28$



13) $y = x^2 - 8x + 15$



14) $y = x^2 - 2x + 5$



TOPIC 40: ARITHMETIC SEQUENCES

Determine if the sequence is arithmetic. If it is, find the common difference, the 52nd term, the explicit formula, and the three terms in the sequence after the last one given.

1) 13, 15, 17, 19, ...

2) 4, 7, 12, 19, ...

3) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

4) 34, 28, 22, 16, ...

Given the explicit formula for an arithmetic sequence find the common difference, the term named in the problem, and the recursive formula.

5) $a_n = 17 + 8n$
Find a_{39}

6) $a_n = -\frac{5}{2} + \frac{3}{2}n$
Find a_{22}

Given two terms in an arithmetic sequence find the common difference, the explicit formula, and the recursive formula.

7) $a_{11} = 110$ and $a_{37} = 370$

8) $a_{10} = 14$ and $a_{37} = 122$

Find the missing terms in each arithmetic sequence.

9) ..., $\frac{3}{2}$, __, __, 0, ...

10) ..., 3.4, __, __, -2, ...

TOPIC 41: GEOMETRIC SEQUENCES

Determine if the sequence is geometric. If it is, find the common ratio.

1) $-1, 6, -36, 216, \dots$

2) $-1, 1, 4, 8, \dots$

3) $4, 16, 36, 64, \dots$

4) $-3, -15, -75, -375, \dots$

5) $-2, -4, -8, -16, \dots$

6) $1, -5, 25, -125, \dots$

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

7) $a_n = 3^{n-1}$

8) $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$

9) $a_n = -2.5 \cdot 4^{n-1}$

10) $a_n = -4 \cdot 3^{n-1}$

Given the recursive formula for a geometric sequence find the common ratio, the first five terms, and the explicit formula.

11) $a_n = a_{n-1} \cdot 2$
 $a_1 = 2$

12) $a_n = a_{n-1} \cdot -3$
 $a_1 = -3$

13) $a_n = a_{n-1} \cdot 5$
 $a_1 = 2$

14) $a_n = a_{n-1} \cdot 3$
 $a_1 = -3$

Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15) $a_1 = 0.8, r = -5$

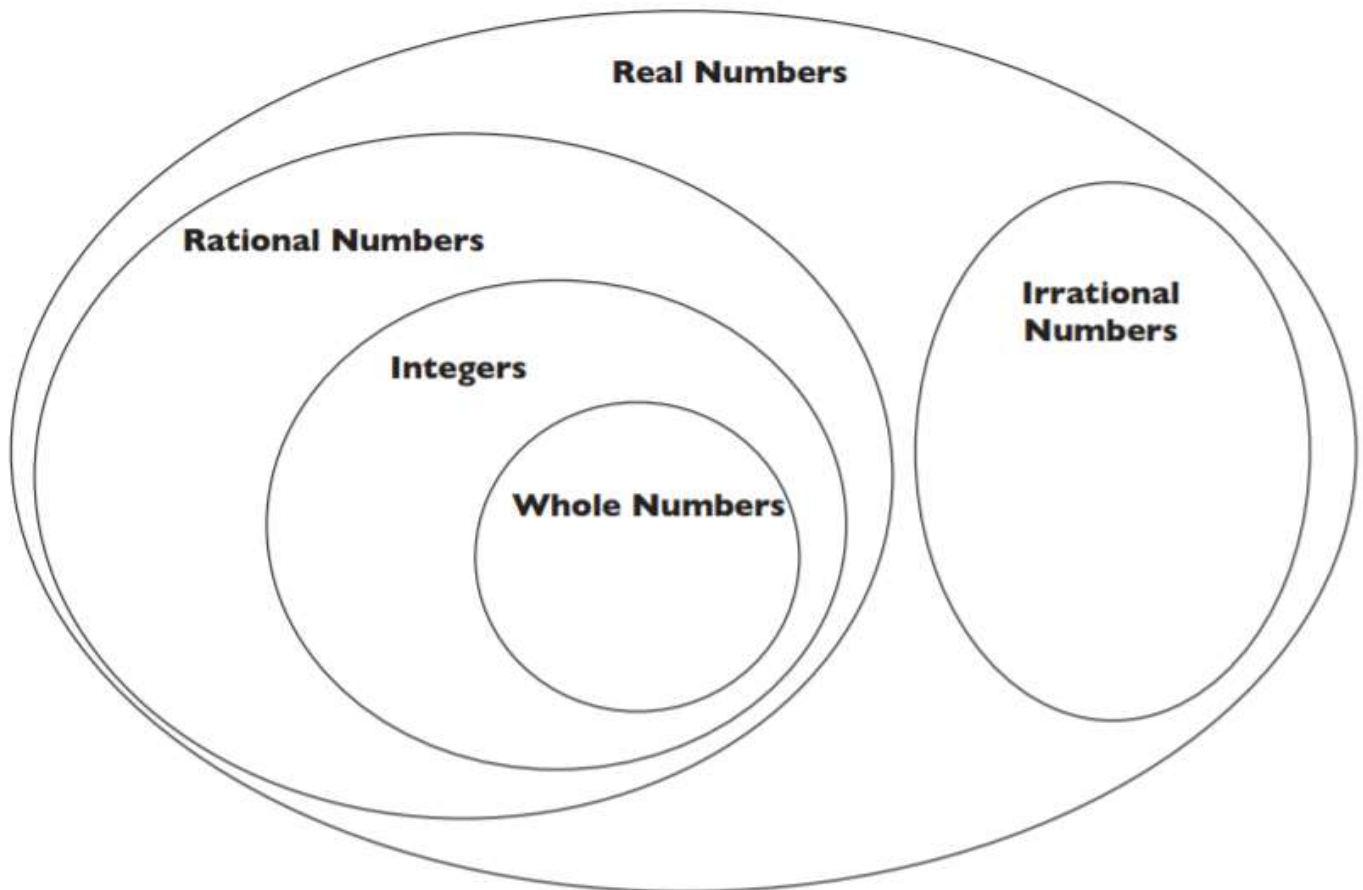
16) $a_1 = 1, r = 2$

TOPIC 42: CLASSIFYING REAL NUMBERS

Directions:

Write each number in the correct location on the Venn Diagram of the real number system. Each number should be written only once.

$$\left(-6, 2.73, \frac{3}{7}, \sqrt{2}, \sqrt{9}, -100, 0, \pi, 1, -\frac{1}{2}, -3.8, 5.\overline{42}, 8.293017... \right)$$



True or false? If false, explain why.

- | | |
|------------------------------------|---|
| 1) All whole numbers are integers. | 3) Some rational numbers are integers. |
| 2) All integers are whole numbers. | 4) Some whole numbers are irrational numbers. |

- 1) List the numbers in the set $\left(\frac{4}{5}, -18, 0, \sqrt{5}, -\frac{1}{2}, -2.01, 5, \pi, \overline{2.513}, 5.1823159\dots\right)$ that are:

Whole numbers

Integers

Rational numbers

Irrational numbers

Real numbers

- 2) Put a check mark for **each set** that the number is a part of:

	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-7					
$\frac{3}{4}$					
$\sqrt{2}$					
5					
0.398					

- 3) True or false? If false, explain why.

- All **integers** are **rational**.
- If a number is **rational**, then it must be a **whole number**.
- Some **irrational numbers** are **integers**.
- All **irrational numbers** are **real numbers**.
- No **whole numbers** are **integers**.